Part (2)

Dynamics

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Chapter (1)

Force and Acceleration

Chapter Objectives

• To state Newton's Second Law of Motion and to define mass and weight.

• To analyze the accelerated motion of a particle u sing the equation of motion with different coordinate systems.

• To investigate central-force motion and apply it to problems in space mechanics.

Newton 's Second Law of Motion

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an ***unbalanced force*** acts on a particle, the particle will ***accelerate*** in the direction of the force with a magnitude that is proportional to the force.

This law can be verified experimentally by applying a known unbalanced force **F** to a particle, and then measuring the acceleration **a**. Since the force and acceleration are directly proportional, the constant of proportionality, ***m***, may be determined from the ratio ***m =F /*** **a**. This positive scalar m is called the ***mass*** of the particle. Being constant during any acceleration, ***m*** provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

If the mass of the particle is m, ***Newton's second law*** of motion may be written in mathematical form as

$$F = ma \quad (1)$$

The above equation, which is referred to as the equation of motion, is one of the most important formulations in mechanics. As previously stated, its validity is based solely on experimental evidence. In 1905, however, Albert Einstein developed the theory of relativity and placed limitations on the use of Newton's second law for describing general particle motion. Through experiments it was proven that time is not an absolute quantity as assumed by Newton; and as a result, the equation of motion fails to predict the exact behavior of a particle, especially when the particle's speed approaches the speed of light (***0.3 Gm/s***). Developments of the theory of quantum mechanics by Erwin Schrodinger and others indicate further that conclusions drawn from using this equation are also invalid when particles are the size of an atom and move close to one another. For the most part, however, these requirements regarding particle speed and size are not encountered in engineering problems, so their effects will not be considered in this book.

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as

$$F = G\frac{m_1 m_2}{r^2} \quad (2)$$

Where:

F = force of attraction between the two particles

G = universal constant of gravitation; according to experimental evidence G = 66.73(10^{-12}) m^3/ (kg. s^2)

m_1, m_2 = mass of each of the two particles

r = distance between the centers of the two particles

In the case of a particle located at or near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the particle. This force is termed the "weight" and, for our purpose, it will be the only gravitational force considered. From Eq. (2), we can develop a general expression for finding the weight ***W*** of a particle having a mass $\boldsymbol{m_1 = m}$. Let $\boldsymbol{m_2 = Me}$ be the mass of the earth and ***r*** the distance between the earth's center and the particle. Then, if $\boldsymbol{g = GMe/r^2}$, we have

W = mg

The Equation of Motion

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e., $\boldsymbol{F_R} = \sum \boldsymbol{F}$. For this more general case, the equation of motion may be written as

$$\sum F = ma \quad (3)$$

To illustrate application of this equation, consider the particle shown in Fig. 1(a), which has a mass m and is subjected to the action of two forces, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$. We can graphically account for the magnitude and direction of each force acting on the particle by drawing the particle's free-body diagram, Fig. 1(b). Since the resultant of these forces produces the vector ***ma***, its magnitude and direction can be represented graphically on the ***kinetic diagram***, shown in Fig. 1(c). The equal sign written between the diagrams symbolizes the graphical equivalency between the free-body diagram and the kinetic diagram; i.e., $\sum \boldsymbol{F} = \boldsymbol{ma}$. In particular, note that if $\boldsymbol{F_R} = \sum \boldsymbol{F} = \boldsymbol{0}$, then the acceleration is also zero, so that the particle will either remain at rest or move along a straight-line path with ***constant velocity***. Such are the conditions of static Newton's first law of motion.

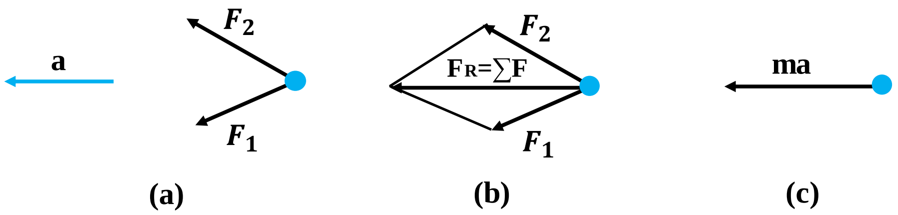


Fig. 1

Equation of Motion for a System of Particles

The equation of motion will now be extended to include a system of particles isolated within an enclosed region in space, as shown in Fig. 2(a). In particular, there is no restriction in the way the particles are connected, so the following analysis applies equally well to the motion of a solid, liquid, or gas system.

At the instant considered, the arbitrary ***i-th*** particle, having a mass m_i, is subjected to a system of internal forces and a resultant external force. The ***internal force***, represented symbolically as $\mathbf{f}_i$, is the resultant of all the forces the other particles exert on the ***ith*** particle. The resultant external force F_i represents, for example, the effect of gravitational, electrical, magnetic, or contact forces between the ***ith*** particle and adjacent bodies or particles not included within the system. The free-body and kinetic diagrams for the ***ith*** particle are shown in Fig. 2(b). Applying the equation of motion,

$$\sum F = ma \qquad F_i + f_i = m_i a_i$$

When the equation of motion is applied to each of the other particles of the system, similar equations will result. And, if all these equations are added together ***vectorially***, we obtain

$$\sum F_i + \sum f_i = \sum m_i a_i$$

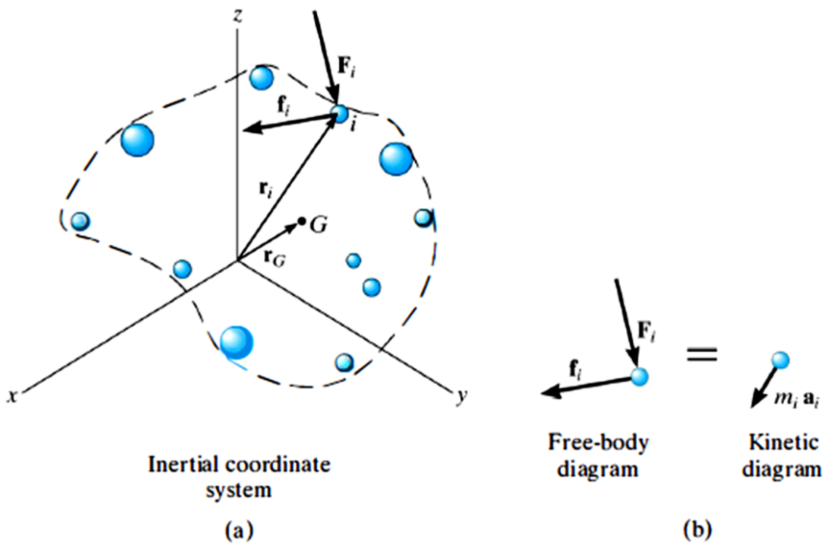


Fig. 2

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\sum F_i = \sum m_i a_i \qquad (4)$$

If r_G is a position vector which locates the center of mass ***G*** of the particles, Fig. 13-4a, then by definition of the center of mass, $\boldsymbol{m r_G} = \sum \boldsymbol{m_i r_i}$, where $\boldsymbol{m} = \sum \boldsymbol{m_i}$ is the total mass of all the particles. Differentiating this equation twice with respect to time, assuming that no mass is entering or leaving the system, yields

$m r_G = \sum m_i r_i$

Substituting this result into Eq. 4, we obtain

$$\sum F = m a_G \quad (5)$$

Hence, the sum of the external forces acting on the system of particles is equal to the total mass of the particles times the acceleration of its center of mass ***G***. Since in reality all particles must have a finite size to possess mass, Eq. 5 justifies application of the equation of motion to a ***body*** that is represented as a single particle.

Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial ***x, y, z*** frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their ***i, j, k*** components, Fig. 3. Applying the equation of motion, we have

$$\sum F = ma; \qquad \sum F_x\, i + \sum F_y\, j + \sum F_z\, k = m\left(a_x i + a_y j + a_z k\right)$$

For this equation to be satisfied, the respective ***i,j, k*** components on the left side must equal the corresponding components on the right side.

Consequently, we may write the following three scalar equations:

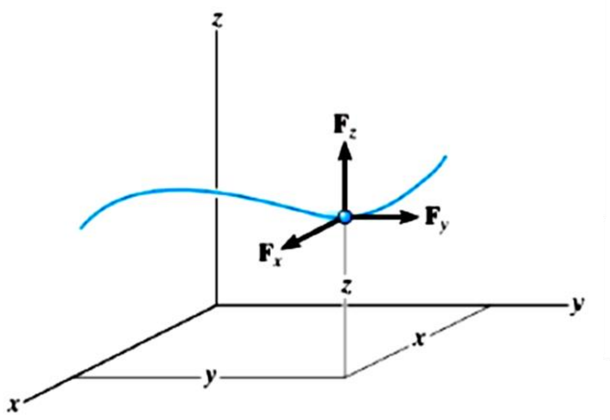


Fig. 3

$$\sum F_x = m a_x$$
$$\sum F_y = m a_y \quad (6)$$
$$\sum F_z = m a_z$$

In particular, if the particle is constrained to move only in the ***x-y*** plane, then the first two of these equations are used to specify the motion.

Example: (1)

The ***50-kg*** crate shown in Fig. 4(a) rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a ***400-N*** towing force as shown, determine the velocity of the crate in 3 s starting from rest.

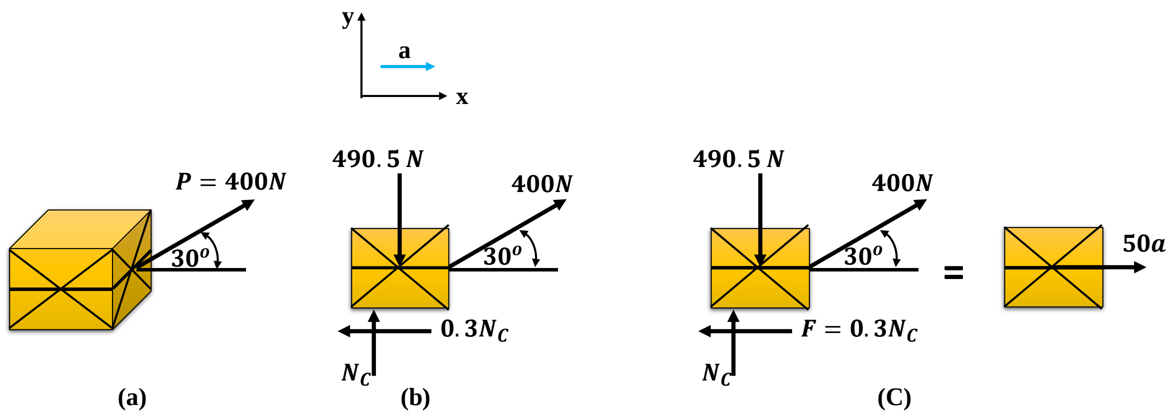


Fig. 4

Solution:

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50\ kg\ (9.81\ m/s^2) = 490.5\ N$. As shown in Fig. 4(b), the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration a is assumed to act horizontally, in the positive ***x direction***. There are two unknowns, namely N_c and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

$$\sum F_x = ma_x ; \qquad 400\cos 30^0 - 0.3N_C = 50a \qquad (1)$$

$$\sum F_y = ma_y ; \qquad N_C - 490.5 + 400\sin 30^0 = 0 \qquad (2)$$

Solving Eq. 2 for ***Nc***, substituting the result into Eq. 1, and solving for a yields

$$N_C = 290.5\ N$$

$$a = 5.185\ m/s^2$$

Kinematics. Notice that the acceleration is constant, Since the applied force ***P*** is constant. Since the initial velocity is zero, the velocity of the crate in ***3 s*** is

$$v = v_0 + a_c t = 0 + 5.185 \times 3 = 15.6\, m/s$$

NOTE: We can also use the alternative procedure of drawing the crate's free-body and kinetic diagrams, Fig. 4(c), prior to applying the equations of motion.

Example: (2)

A smooth **2-kg** collar C, shown i n Fig. 5(a), i s attached t o a spring having a stiffness ***k* = 3 *N/m*** and an unstretched length of ***0.75 m***. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant ***y* = *1 m***.

Solution:

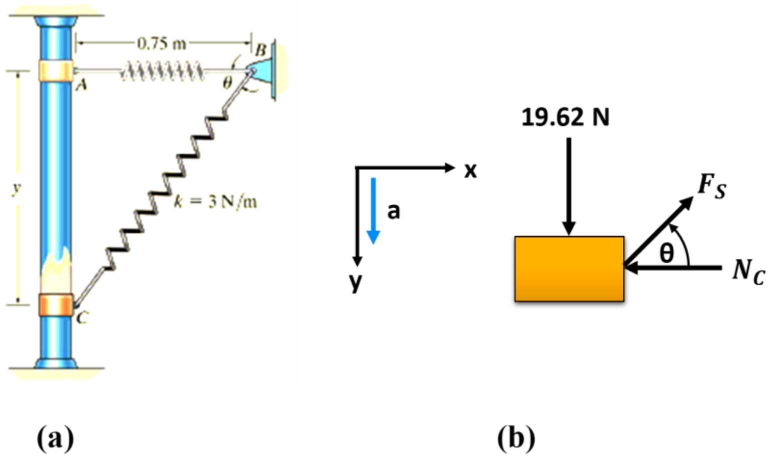


Fig. 5

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position y is shown in Fig. 5(b). Furthermore, the collar is assumed to be accelerating so that "**a**" act downward in the positive ***y*** direction. There are four unknowns, namely, ***Nc***, ***Fs***, **a**, and ***θ***.

Equations of Motion.

$$\rightarrow \sum F_x = ma_x; \qquad -N_C + F_S \cos\theta = 0 \qquad (1)$$

$$\downarrow \sum F_y = ma_y; \qquad 19.62 + F_S \sin\theta = 2a \qquad (2)$$

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for ***Nc*** and **a** is possible once ***Fs*** and ***θ*** are known.

The magnitude of the spring force is a function of the stretch s of the spring; i.e., $\boldsymbol{F_s = ks}$. Here the unstretched length is $\boldsymbol{AB = 0.75\ m}$, Fig. 5(a); therefore, $\boldsymbol{s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75}$. Since ***k* = *3 N/m***, then

$$F_s = ks = 3\left[\sqrt{y^2 + (0.75)^2} - 0.75\right] \qquad (3)$$

From Fig. 5(a), the angle ***θ*** is related to y by trigonometry.

$$\tan\theta = \frac{y}{0.75} \qquad (4)$$

Substituting ***y* = *1 m*** into Eqs. (3) and (4) yields ***Fs* = *1.50 N*** and ***θ* = *53.1°***. Substituting these results into Eqs. (1) and (2), we obtain

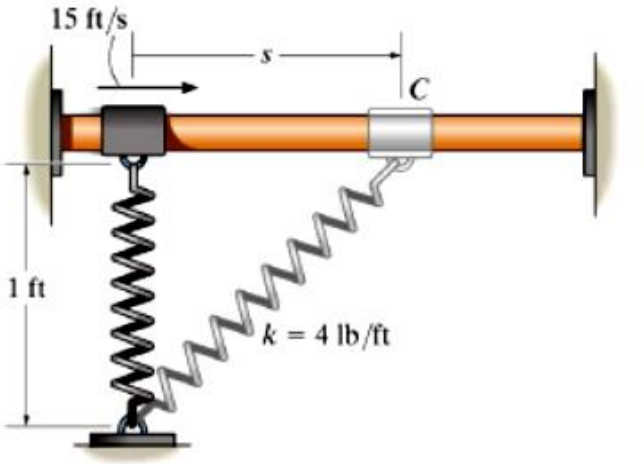
$$N_C = 0.900\ N$$

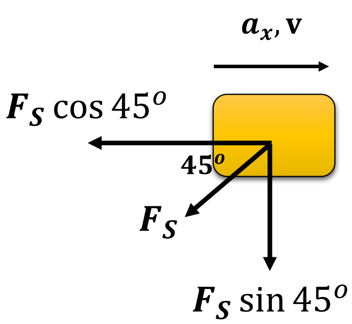
$$a = 9.21\ m/s^2 \ \downarrow$$

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.

Solved Example

1. The ***2-lb*** collar C fits loosely on the smooth shaft. If the spring is unstretched when ***s = 0*** and the collar is given a velocity of ***15 ft/s***, determine the velocity of the collar when ***s = 1 ft***.





Free body diagram

Data:

$v_0 = 15\ ft/s$, s = 1 ft, k = 4 Ib/ft, m = 2 Ib and l_o = 1 ft

Req. v = ??

Solution:

$\sum F_x = ma_x$

$v = v_0 + a_x t$

$S = S_0 + \frac{1}{2} a_x t^2$

$v^2 = v_0^2 + 2a_x S$

$$F_s = spring\ force = k \times \Delta l$$

$$\Delta l = l_f - l_o$$

$$l_f = \sqrt{1^2 + 1^2} = \sqrt{2} = 1.41\ ft$$

$$\therefore \Delta l = 1.41 - 1 = 0.41\ ft$$

$$F_s = 4 \times 0.41 = 1.64\ Ib$$

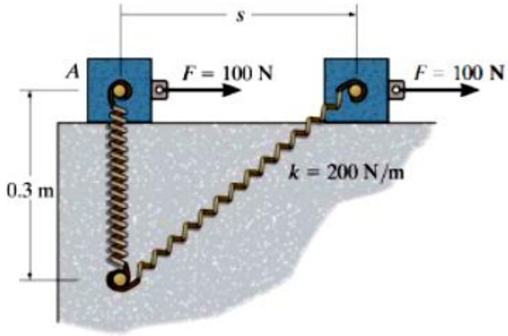
$$F_S \cos 45^o = ma_x$$

$$\therefore a_x = \frac{F_S \cos 45^o}{m} = \frac{1.64 \cos 45^o}{2} = 0.58\ ft/s^2$$

$\because v^2 = v_0^2 + 2a_x S$

$$\therefore v = \sqrt{v_0^2 + 2a_x S} = \sqrt{15^2 + 2 \times 0.58 \times 1} = 15.04\ ft/s$$

2. The spring has a stiffness ***k = 200 N/m*** and is unstretched when the ***25-kg*** block is at A. Determine the acceleration of the block when ***s = 0.4 m***. The contact surface between the block and the plane and the coeff. of friction μ = 0.3.

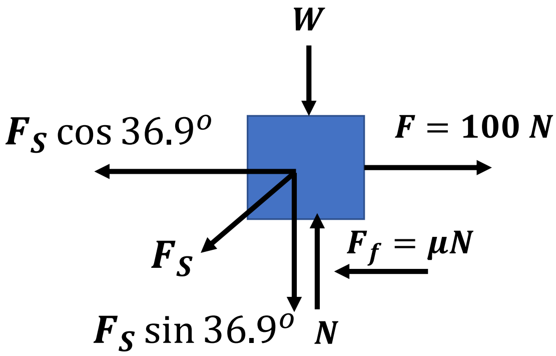


Data: F = 100 N, k = 200 N/m, m = 25 kg, S = 0.4 m and μ = 0.3

Req.: a = ??

Solution:

$\sum F_x = ma_x$



Free body diagram

$$F_S = k\Delta l$$

$$\Delta l = l_f - l_o$$

$$l_f = \sqrt{0.3^2 + 0.4^2} = 0.5\ m$$

$$\therefore \Delta l = 0.5 - 0.3 = 0.2\ m$$

$$\therefore F_s = 200 \times 0.2 = 40\ N$$

$\sum F_y = ma_y = 0$

$$-W - F_S \sin 36.9 + N = 0$$

$$N = W + F_S \sin 36.9 = (25 \times 9.81) + (40 \sin 36.9) = 269.3\ N$$

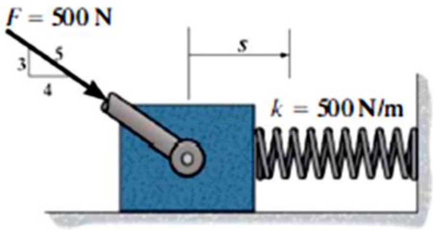
$$\therefore F_f = friction\ force = \mu N = 0.3 \times 269.3 = 80.8\ N$$

$\sum F_x = ma_x$

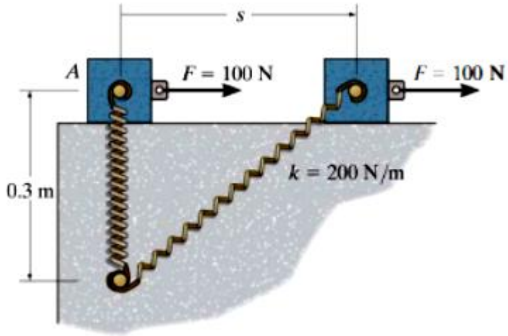
$a_x = \frac{\sum F_x}{m} = \frac{100 - F_S \cos 36.9 - F_f}{m} = \frac{100 - 40\cos 36.9 - 80.8}{25} = -\overrightarrow{0.511}\ m/s^2 = \overleftarrow{0.511}\ m/s^2$

Fundamental Problems

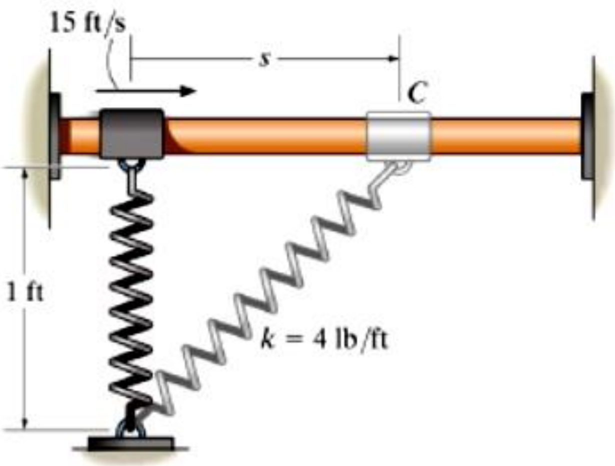
3. A spring of stiffness ***k = 500 N/m*** is mounted against the ***10-kg*** block. If the block is subjected to the force of ***F = 500 N***, determine its velocity at ***s = 0.5 m***. When ***s = 0***, the block is at rest and the spring is uncompressed. The contact surface is smooth.



4. The spring has a stiffness ***k = 200 N/m*** and is unstretched when the ***25-kg*** block is at A. Determine the acceleration of the block when ***s = 0.4 m***. The contact surface between the block and the plane is smooth.



5. The ***2-lb*** collar C fits loosely on the smooth shaft. If the spring is unstretched when ***s = 0*** and the collar is given a velocity of ***15 ft/s***, determine the velocity of the collar when ***s = 1 ft***.



Equations of Motion: Normal and Tangential Coordinates

When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binormal directions, Fig. 6. Note that there is no motion of the particle in the binormal direction, since the particle is constrained to move along the path. We have

$$\sum F = ma; \qquad \sum F_t u_t + \sum F_n u_n + \sum F_b u_b = ma_t + ma_n$$

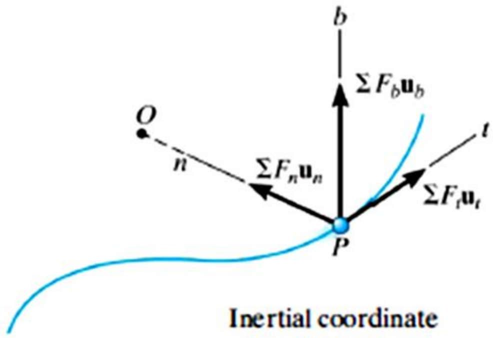


Fig. 6

This equation is satisfied provided

$$\sum F_t = ma_t$$

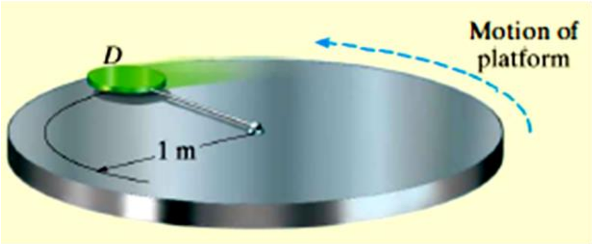
$$\sum F_n = ma_n \qquad (7)$$

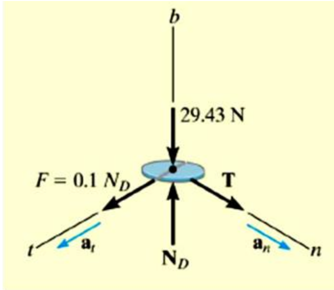
$$\sum F_b = 0$$

Recall that $\mathbf{a_t}$ *(= dv/ dt)* represents the time rate of change in the magnitude of velocity. So if $\sum \boldsymbol{F_t}$ acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise, $\mathbf{a_n}$ *(=* v^2/ρ*)* represents the time rate of change in the velocity's direction. It is caused by $\sum \boldsymbol{F_n}$, which *always* acts in the positive ***n*** direction, i.e., toward the path's center of curvature. From this reason it is often referred to as the ***centripetal force***.

Example: (3)

The ***3-kg*** disk D is attached to the end of a cord as shown in Fig. 7(a). The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is ***100 N***, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.





(a) (b)

Fig. 7

Solution:

Free-Body Diagram. The frictional force has a magnitude $F = \mu_k N_D = 0.1 N_D$ and a sense of direction that opposes the relative of the disk with respect to the platform. It is this force that gives the disk a tangential component of acceleration causing v to increase, thereby causing T to increase until it reaches $100\ N$. The weight of the disk is $W = 3(9.81) = 29.43\ N$. Since $\mathbf{a}_n$ can be related to v, the unknowns are N_D, $\mathbf{a}_t$, and v.

Equations of Motion.

$$\sum F_n = ma_n; \qquad T = 3\left(\frac{v^2}{1}\right) \qquad (1)$$

$$\sum F_t = ma_t; \qquad 0.1N_D = 3a_t \qquad (2)$$

$$\sum F_b = 0; \qquad N_D - 29.43 = 0 \qquad (3)$$

Setting $T = 100\ N$, Eq. (1) can be solved for the critical speed v_{cr} of the disk needed to break the cord. Solving all the equations, we obtain

$$N_D = 29.43$$

$$a_t = 0.981\, m/s^2$$

$$v_{cr} = 5.77\, m/s$$

Kinematics. Since $\mathbf{a}_t$ is constant, the time needed to break the cord is

$$v_{cr} = v_0 + a_t t$$

$$\therefore 5.77 = 0 + (0.981)t$$

$$\therefore t = 5.89\ s$$

Central-Force Motion and Space Mechanics

If a particle is moving only under the influence of a force having a line of action which is always directed toward a fixed point, the motion is called central-force motion. This type of motion is commonly caused by electrostatic and gravitational forces.

$$\sum F_r = ma_r$$
$$\sum F_\theta = ma_\theta \qquad (8)$$
$$\sum F_z = ma_z$$

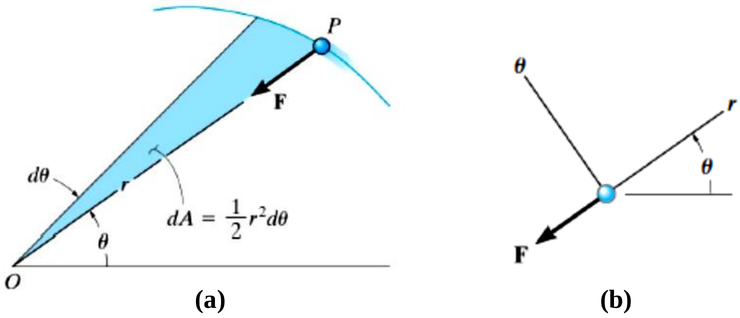


Fig. 8

In order to analyze the motion, we will consider the particle ***P*** shown in Fig. 8(a), which has a mass ***m*** and is acted upon only by the central force ***F***. The free-body diagram for the particle is shown in Fig. 8(b). Using polar coordinates (***r, θ***), the equations of motion, Eqs. 8, become

$$\sum F_r = ma_r \qquad \therefore -F = m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]$$

$$(9)$$

$$\sum F_\theta = ma_\theta \qquad \therefore 0 = m\left(r\frac{d^2r}{dt^2} - 2\frac{dr}{dt}\frac{d\theta}{dt}\right)$$

The second of these equations may be written in the form

$$\frac{1}{r}\left[\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right] = 0$$

so that integrating yields

$$r^2\frac{d\theta}{dt} = h \qquad (10)$$

Here ***h*** is the constant of integration.

From Fig. 8(a) notice that the shaded area described by the radius ***r***, as ***r*** moves through an angle ***dθ***, is ***dA=*** r^2***dθ***. If the areal velocity is defined as

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{h}{2} \qquad (11)$$

then it is seen that the areal velocity for a particle subjected to central force motion is constant. In other words, the particle will sweep out equal segments of area per unit of time as it travels along the path. To obtain the path of motion, ***r = f(θ)***, the independent variable t must be eliminated from Eqs. (9). Using the chain rule of calculus and Eq. (10), the time derivatives of Eqs. (9) may be replaced by

$$\frac{dr}{dt} - \frac{dr}{d\theta}\frac{d\theta}{dt} - \frac{h}{r^2}\frac{dr}{d\theta}$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{h}{r^2}\frac{dr}{d\theta}\right) = \frac{d}{d\theta}\left(\frac{h}{r^2}\frac{dr}{d\theta}\right)\frac{d\theta}{dt} = \left[\frac{d}{d\theta}\left(\frac{h}{r^2}\frac{dr}{d\theta}\right)\right]\frac{h}{r^2}$$

Substituting a new dependent variable (**xi**) $\xi = l/r$ into the second equation, we have

$$\frac{d^2r}{dt^2} = -h^2\xi^2\frac{d^2\xi}{d\theta^2}$$

Also, the square of Eq. (10) becomes

$$\left(\frac{d\theta}{dt}\right)^2 = h^2\xi^4$$

Substituting these two equations into the first of Eqs. (9) yields

$$-h^2\xi^2\frac{d^2\xi}{d\theta^2} - h^2\xi^3 = -\frac{F}{m}$$

Or

$$\frac{d^2\xi}{d\theta^2} + \xi = \frac{F}{mh^2\xi^2} \qquad (12)$$

This differential equation defines the path over which the particle travels when it is subjected to the central force ***F***.

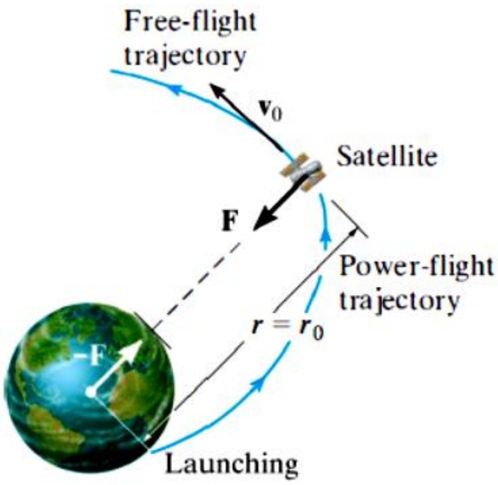


Fig. 9

For application, the force of gravitational attraction will be considered. Some common examples of central-force systems which depend on gravitation include the motion of the moon and artificial satellites about the earth, and the motion of the planets about the sun. As a typical problem in space mechanics, consider the trajectory of a space satellite or space vehicle launched into free-flight orbit with an initial velocity $\boldsymbol{V_0}$, Fig. 9. It will be assumed that this velocity is initially parallel to the tangent at the surface of the earth, as shown in the figure.t Just after the satellite is released into free flight, the only force acting on it is the gravitational force of the earth. (Gravitational attractions involving other bodies such as the moon or sun will be neglected, since for orbits close to the earth their effect is small in comparison with the earth's gravitation.) According to Newton's law of gravitation, force $\boldsymbol{F}$ will always act between the mass centers of the earth and the satellite, Fig. 9. From Eq. (2), this force of attraction has a magnitude of

$$F = G\frac{M_e m}{r^2}$$

Where: $\boldsymbol{M_e}$ and $\boldsymbol{m}$ represent the mass of the earth and the satellite, respectively, $\boldsymbol{G}$ is the gravitational constant, and $\boldsymbol{r}$ is the distance between the mass centers. To obtain the orbital path, we set $\boldsymbol{\xi = 1/r}$ in the foregoing equation and substitute the result into Eq. (12). We obtain

$$\frac{d^2\xi}{d\theta^2} + \xi = \frac{GM_e}{h^2} \qquad (13)$$

This second-order differential equation has constant coefficients and is nonhomogeneous. The solution is the sum of the complementary and particular solutions given by

$$\xi = \frac{1}{r} = C\cos(\theta - \phi) + \frac{GM_e}{h^2} \qquad (14)$$

This equation represents the ***free-flight trajectory*** of the satellite. It is the equation of a conic section expressed in terms of polar coordinates.

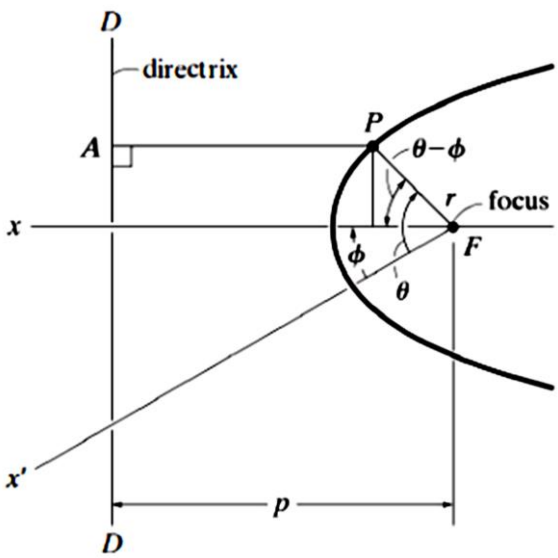


Fig. 10

A geometric interpretation of Eq. (14) requires knowledge of the equation for a conic section. As shown in Fig. 10, a conic section is defined as the locus of a point ***P*** that moves in such a way that the ratio of its distance to a ***focus***, or fixed-point ***F***, to its perpendicular distance to a fixed line ***DD*** called the ***directrix***, is constant. This constant ratio will be denoted as e and is called the ***eccentricity***. By definition

$$e = \frac{FP}{PA}$$

From Fig. 10,

$$FP = r = e(PA) = e[p - r\cos(\theta - \phi)]$$

Or

$$\frac{1}{r} = \frac{1}{p}\cos(\theta - \phi) + \frac{1}{ep}$$

Comparing this equation with Eq. 14, it is seen that the fixed distance from the focus to the directrix is

$$p = \frac{1}{C} \tag{15}$$

And the eccentricity of the conic section for the trajectory is

$$e = \frac{Ch^2}{GM_e} \tag{16}$$

Provided the polar angle $\boldsymbol{\theta}$ is measured from the ***x*** axis (an axis of directrix symmetry since it is perpendicular to the directrix), the angle $\boldsymbol{\phi}$ is zero, Fig. 10, and therefore Eq. (14) reduces to

$$\frac{1}{r} = C\cos\theta + \frac{GM_e}{h^2} \tag{17}$$

The constants ***h*** and ***C*** are determined from the data obtained for the positon and velocity of the satellite at the end of the ***power-flight trajectory***. For example, if the initial height or distance to the space vehicle is $\boldsymbol{r_o}$, measured from the center of the earth, and its initial speed is $\boldsymbol{v_o}$ at the beginning of its free flight, Fig. 11, then the constant ***h*** may be obtained from Eq. 10. When $\boldsymbol{\theta = \phi = 0°}$, the velocity Vo has no radial component; therefore, $\boldsymbol{v_0 = r_0(d\theta/dt)}$, so that

$$h = r_0^2\frac{d\theta}{dt}$$

or

$$h = r_0 v_0 \tag{18}$$

To determine ***C***, use Eq. (17) with $\boldsymbol{\theta = 0°}$, $\boldsymbol{r = r_o}$, and substitute Eq. (18) for ***h***:

$$C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \tag{19}$$

The equation for the free-flight trajectory therefore becomes

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\cos\theta + \frac{GM_e}{r_0^2 v_0^2} \quad (20)$$

The type of path traveled by the satellite is determined from the value of the eccentricity of the conic section as given by Eq. 16. If

$$\begin{aligned} &e = 0 \quad free - flight\ trajectory\ is\ a\ circle \\ &e = 1 \quad free - flight\ trajectory\ is\ a\ parabola \\ &e < 1 \quad free - flight\ trajectory\ is\ an\ ellipse \\ &e > 1 \quad free - flight\ trajectory\ is\ a\ hyparabola \end{aligned} \quad (21)$$

Parabolic Path. Each of these trajectories is shown in Fig. 11. From the curves it is seen that when the satellite follows a parabolic path, it is "on the border" of never returning to its initial starting point. The initial launch velocity, $\boldsymbol{v_o}$, required for the satellite to follow a parabolic path is called the escape velocity. The speed, $\boldsymbol{v_e}$, can be determined by using the second of Eqs. (21), ***e = 1***, with Eqs. (16), (18), and (19). It is left as an exercise to show that

$$v_e = \sqrt{\frac{2GM_e}{r_0}} \quad (22)$$

Circular Orbit. The speed $\boldsymbol{v_c}$ required to launch a satellite into a circular orbit can be found using the first of Eqs. (21), ***e = 0***. Since ***e*** is related to ***h*** and ***C***, Eq. (16), ***C*** must be zero to satisfy this equation (from Eq. (18), ***h*** cannot be ***zero***); and therefore, using Eq. (19), we have

$$v_e = \sqrt{\frac{GM_e}{r_0}} \quad (23)$$

Provided $\boldsymbol{r_o}$ represents a minimum height for launching, in which frictional resistance from the atmosphere is neglected, speeds at launch which are less than $\boldsymbol{v_c}$ will cause the satellite to reenter the earth's atmosphere and either burn up or crash, Fig. 11.

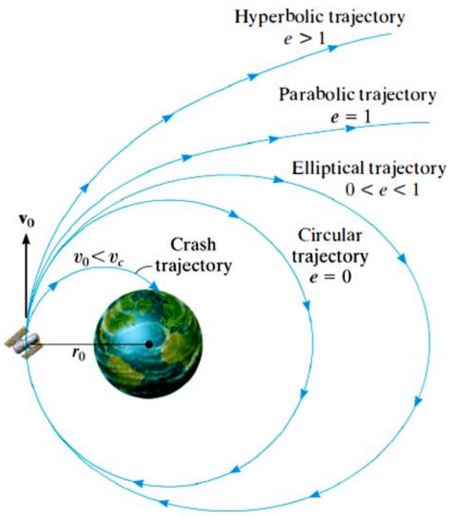


Fig. 11

Elliptical Orbit. All the trajectories attained by planets and most satellites are elliptical, Fig. 12. For a satellite's orbit about the earth, the minimum distance from the orbit to the center of the earth ***0*** (which is located at one of the foci of the ellipse) is r_p and can be found using Eq. 13-22 with ***θ = 0°***. Therefore;

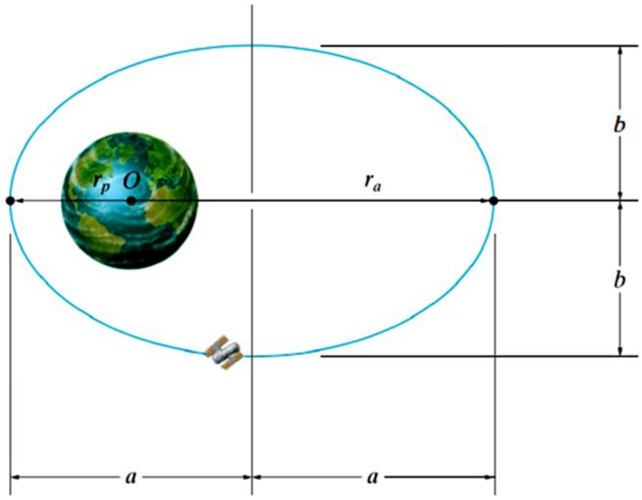


Fig. 12

$$r_P = r_0 \tag{24}$$

This minimum distance is called the perigee of the orbit. The apogee or maximum distance r_a can be found using Eq. (20) with ***θ = 1 80°***. Thus,

$$r_P = \frac{r_0}{(2GM_e/r_0 v_0^2) - 1} \tag{25}$$

With reference to Fig. 12, the half-length of the major axis of the ellipse is

$$a = \frac{r_P + r_a}{2} \tag{26}$$

Using analytical geometry, it can be shown that the half length of the minor axis is determined from the equation

$$b = \sqrt{r_P r_a} \tag{27}$$

Furthermore, by direct integration, the area of an ellipse is

$$A = \pi ab = \frac{\pi}{2}(r_P + r_a)\sqrt{r_P r_a} \tag{28}$$

The areal velocity has been defined by Eq. (11), ***dA/dt = h/2***. Integrating yields, ***A = hT /2***, where ***T*** is the period of time required to make one orbital revolution. From Eq. (28), the period is

$$T = \frac{\pi}{h}(r_P + r_a)\sqrt{r_P r_a} \tag{29}$$

Example: (4)

A satellite is launched ***600 km*** from the surface of the earth, with an initial velocity of ***30 Mm/h*** acting parallel to the tangent at the surface of the earth, Fig. 13. Assuming that the radius of the earth is ***6378 km*** and that its mass is ***5.976(10^{24}) kg***, determine (a) the eccentricity of the orbital path, and (b) the velocity of the satellite at apogee.

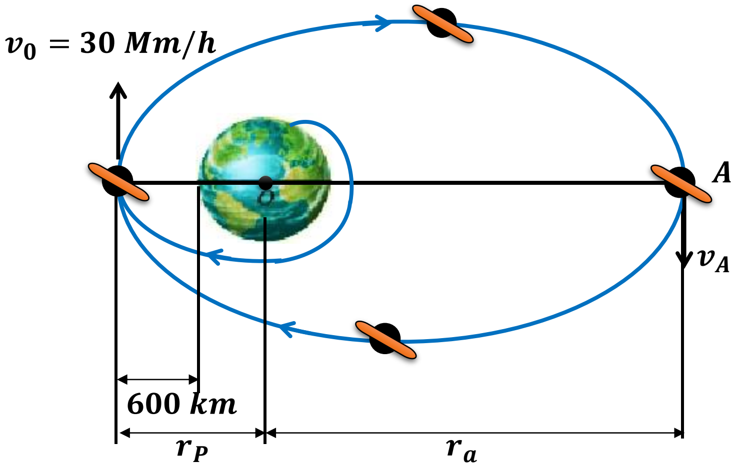


Fig. 13

Solution:

Part (a). The eccentricity of the orbit is obtained using Eq. (16). The constants ***h*** and ***C*** are first determined from Eqs. (18) and (19). Since

$r_p = r_0 = 6378\text{ km} + 600\text{ km} = 6.978(10^6)\text{ m}$

$v_o = 30\text{ Mm/h} = 8333.3\text{ m/s}$

$h = r_p v_o = 6.978(10^6)\,(8333.3) = 58.15(10^9)\text{ m}^2\text{/s}$

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P v_0^2}\right) = \frac{1}{6.978(10^6)}\left\{1 - \frac{66.73(10^{-12})[5.976(10^{24})]}{6.978(10^6)(8333.3)^2}\right\} = 25.4(10^{-9})m^{-1}$$

Hence,

$$e = \frac{Ch^2}{GM_e} = \frac{2.54(10^{-8})[58.15(10^9)]^2}{66.73(10^{-12})[5.976(10^{24})]} = 0.215 < 1$$

From Eq. (20), observe that the orbit is an ***ellipse***.

Part (b). If the satellite were launched at the apogee ***A*** shown in Fig. 13, with a velocity $\boldsymbol{v_A}$, the same orbit would be maintained provided

$h = r_p.v_o = r_a.v_A = 58.15\,(10^9)\text{ m}^2\text{/s}$

Using Eq. (25), we have

$$r_a = \frac{r_P}{(2GM_e/r_P v_0^2) - 1} = \frac{6.978(10^6)}{\frac{2[66.73(10^{-12})][5.976(10^{24})]}{6.978(10^6)(8333.3)^2} - 1} = 10.804(10^6)$$

Thus,

$$v_A = \frac{h}{r_a} = \frac{58.15\ (10^9)}{10.804(10^6)} = 5382.2\ m/s = 19.4\ Mm/h$$

NOTE: The farther the satellite is from the earth, the slower it moves, which is to be expected since $\boldsymbol{h}$ is constant.

Chapter (2)

Work and Energy

CHAPTER OBJECTIVES

• To deve lop the principle of work and energy and apply it to solve problems that involve force, velocity, and displacement.

• To study p roblems that involve power and efficiency.

• To introduce the concept of a conservative force and apply the theorem of conservation of energy to solve kinetic probl ems.

The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this; however, we must first define the work of a force. Specifically, a force ***F*** will do work on a particle only when the particle undergoes a displacement in the ***direction of the force***. For example, if the force ***F*** in Fig. 14 causes the particle to move along the path ***s*** from position ***r*** to a new position ***r'***, the displacement is then ***dr = r' - r***. The magnitude of ***dr*** is ***ds***, the length of the differential segment along the path. If the angle between the tails of ***dr*** and ***F*** is ***θ***, Fig. 14, then the work done by ***F*** is a scalar quantity, defined by

$$dU = F ds \cos\theta$$

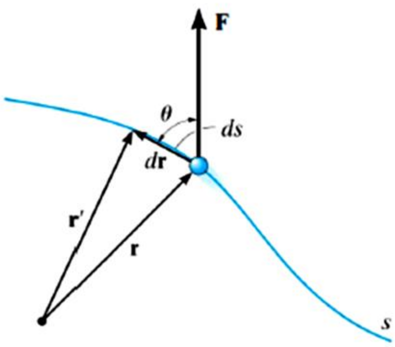


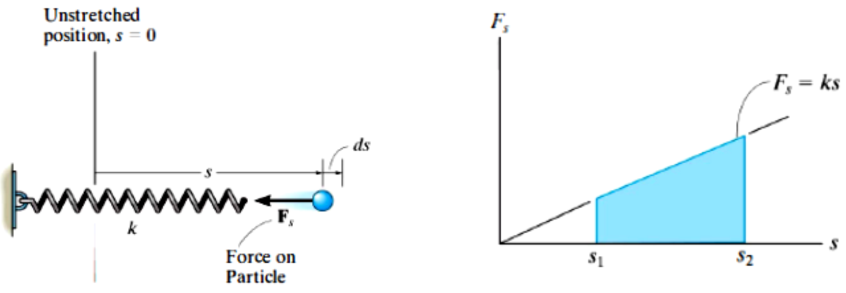
Fig. 14

Work of a Spring Force.

If an elastic spring is elongated a distance ds, Fig. 15(a), then the work done by the force that acts on the attached particle is $dU = -F_s ds = -k_s\, ds$. The work is negative since F_s acts in the opposite sense to ***ds***. If the particle displaces from S_l to S_2, the work of F_s is then

$$U_{1-2} = \int_{S1}^{S2} F_S ds = \int_{S1}^{S2} -k_S ds$$

$$\therefore U_{1-2} = -\left[\frac{1}{2}k_{S2}^2 - \frac{1}{2}k_{S1}^2\right] \qquad (30)$$



(a) (b)

Fig. 15

This work represents the trapezoidal area under the line $\boldsymbol{F_s = kS}$, Fig. 15(b).

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle if both are in the same sense, positive work results; if they are opposite to one another, the work is negative.

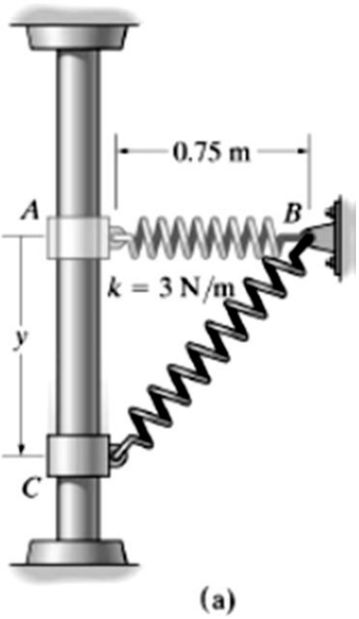
Conservative Forces and Potential Energy

Conservative Force. If the work of a force is independent of the path and depends only on the force's initial and final positions on the path, then we can classify this force as a conservative force. Examples of conservative forces are the weight of a particle and the force developed by a spring. The work done by the weight depends only on the vertical of the weight, and the work done by a spring force depends only on the spring's elongation or compression.

In contrast to a conservative force, consider the force of friction exerted on a sliding object by a fixed surface. The work done by the frictional force depends on the path-the longer the path, the greater the work. Consequently, ***frictional forces are non-conservative***. The work is dissipated from the body in the form of heat.

Example: (5)

A smooth ***2-kg*** collar, shown in Fig. 14(a), fits loosely on the vertical shaft. If the spring is unstretched when the collar is in the position ***A***, determine the speed at which the collar is moving when ***y = 1 m***, if (a) it is released from rest at ***A***, and (b) it is released at ***A*** with an upward velocity $v_A = 2\ m/s$.



(a)

Solution:

(a) It is released from rest at A

At point A:

$v_A = 0,\ \Delta x_A = 0,\ h_A = y = 1$

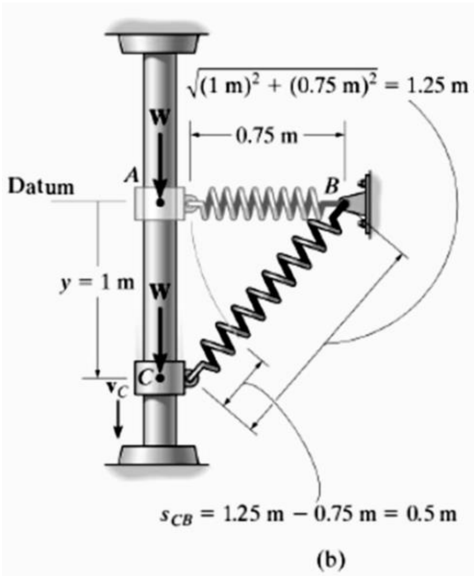
Kinetic Energy T_A

$$\therefore T_A = \frac{1}{2} m v_A^2 = 0$$

Potential Energy U_A

$$\therefore U_A = \frac{1}{2} k \Delta x_A^2 + m g h_A = 0 + 2 \times 9.81 \times 1 = 19.62\ N.m = 19.62\ J$$

At point C:



(b)

$v_C = ??,\ h_C = 0$

$$\Delta x_C = L_f - L_0 = \sqrt{1^2 + 0.75^2} - 0.75 = 0.5\ m$$

Kinetic Energy T_C

$$\therefore T_C = \frac{1}{2} m v_C^2 = \frac{1}{2} 2 v_C^2 = v_C^2$$

Potential Energy U_C

$$\therefore U_C = \frac{1}{2} k \Delta x_C^2 + m g h_C = \frac{1}{2} 3 \times 0.5^2 + 0$$
$$= \frac{3}{8}\ N.m = 0.375\ N.m = 0.375\ J$$

By using Conservation of Energy

$$T_A + U_A = T_C + U_C$$

$$\therefore 0 + 19.62 = v_C^2 + 0.375$$

$$v_C = \sqrt{19.62 - 0.375} = 4.39\ m/s$$

(b) it is released at A with an upward velocity $v_A = 2\ m/s$

At point A:

$v_A = 2\ m/s,\ \Delta x_A = 0,\ h_A = y = 1$

Kinetic Energy T_A

$$\therefore T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}2 \times 2^2 = 4\ N.m = 4\ J$$

Potential Energy U_A

$$\therefore U_A = \frac{1}{2}k\Delta x_A^2 + mgh_A = 0 + 2 \times 9.81 \times 1 = 19.62\ N.m = 19.62\ J$$

At point C:

$v_C = ??,\ h_C = 0$

$\Delta x_C = L_f - L_0 = \sqrt{1^2 + 0.75^2} - 0.75 = 0.5\ m$

Kinetic Energy T_C

$$\therefore T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}2v_C^2 = v_C^2$$

Potential Energy U_C

$$\therefore U_C = \frac{1}{2}k\Delta x_C^2 + mgh_C = \frac{1}{2}3 \times 0.5^2 + 0 = \frac{3}{8}\ N.m = 0.375\ N.m = 0.375\ J$$

By using Conservation of Energy

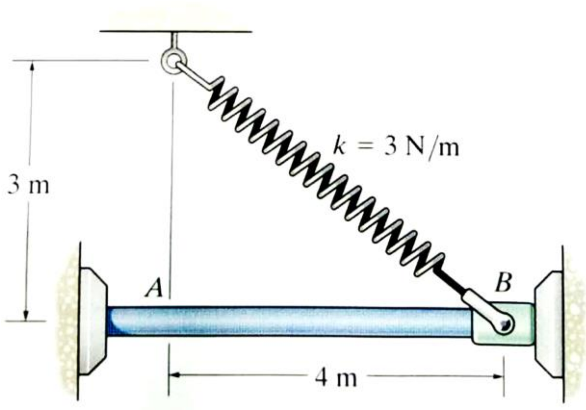
$T_A + U_A = T_C + U_C$

$\therefore 4 + 19.62 = v_C^2 + 0.375$

$$v_C = \sqrt{19.62 + 4 - 0.375} = 4.821\ m/s$$

Example: (6)

The 2-kg collar is attached to a spring that has an unstretched length of 2 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



Solution:

Data:

$L_0 = 2\ m,\ k = 3\ N/m,\ v_B = 0$

At point B:

$v_B = 0\ ,\ \Delta x_B = 5 - 2 = 3\ m,\ h_B = 0$

Kinetic Energy T_B

$$\therefore T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}2\times 0^2 = 0$$

Potential Energy U_B

$$\therefore U_B = \frac{1}{2}k\Delta x_B^2 + mgh_B = \frac{1}{2}3\times 3^2 + 0 = 13.5\,N.m = 13.5\,J$$

At point A:

$v_A = ??$, $\Delta x_A = 3 - 2 = 1\,m$, $h_A = 0$

Kinetic Energy T_A

$$\therefore T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}2\times v_A^2 = v_A^2$$

Potential Energy U_A

$$\therefore U_A = \frac{1}{2}k\Delta x_A^2 + mgh_A = \frac{1}{2}3\times 1^2 + 0 = 1.5\,N.m = 1.5\,J$$

By using Conservation of Energy

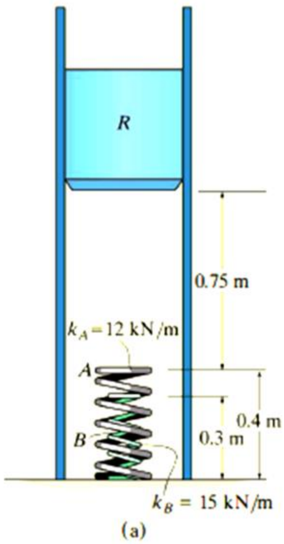
$T_A + U_A = T_B + U_B$

$\therefore v_A^2 + 1.5 = 0 + 13.5$

$$v_A = \sqrt{13.5 - 1.5} = 3.464\,m/s$$

Example: (7)

The ram ***R*** shown in Fig. a has a mass of ***100 kg*** and is released from rest ***0.75 m*** from the top of a spring, A, that has a stiffness ***k_A = 12 kN/m***. If a second spring B, having a stiffness ***k_B = 15 kN/m***, is "nested" in ***A***, determine the maximum displacement of ***A*** needed to stop the downward motion of the ram. The unstretched length of each spring is indicated in the figure. Neglect the mass of the springs.



(a)

Solution:

Potential Energy. We will assume that the ram compresses both springs at the instant it comes to rest. The datum is located through the center of gravity of the ram at its initial position, Fig. b. When the kinetic energy is reduced to zero (**V_2 = 0**), **A** is compressed a distance **S_A** and **B** compresses **S_B = S_A - 0.1 m**.

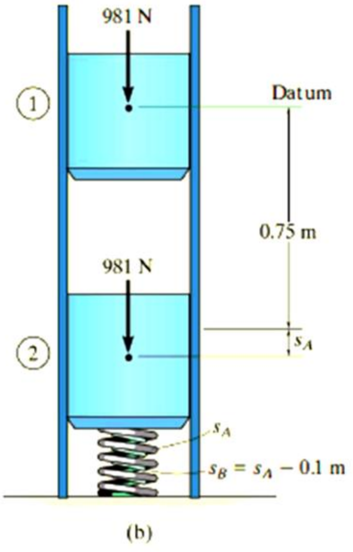
At position (1)

$T_1 = 0$ at $v_1 = 0$

$V_1 = 0$ at $h_1 = 0$

At position (2)

$T_2 = 0$ at $v_2 = 0$



$$V_2 = \frac{1}{2}k_A S_A^2 + \frac{1}{2}k_B S_B^2 - mgh_2 = \frac{1}{2}k_A S_A^2 + \frac{1}{2}k_B(S_A - 0.1)^2 - Wh_2$$

$$= \left(\frac{1}{2}12000 \times S_A^2\right) + \left(\frac{1}{2}15000(S_A - 0.1)^2\right) - \left(981(0.75 + S_A)\right)$$

By using the Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \left(\frac{1}{2}12000 \times S_A^2\right) + \left(\frac{1}{2}15000(S_A - 0.1)^2\right) - \left(981(0.75 + S_A)\right)$$

Rearranging the terms,

$$13500\, S_A^2 - 2481\, S_A - 660.75 = 0$$

Using the quadratic formula and solving for the positive root, we have

$S_A = 0.331$ m **Ans.**

Since $S_B = 0.331$ m - 0.1 m = 0.231 m, which is positive, the assumption that both springs are compressed by the ram is correct.

Chapter (3)

Impulse and Momentum

CHAPTER OBJECTIVES

- To develop the principle of linear impulse and momentum for a particle and apply it to solve problems that involve force, velocity, and time.
- To study the conservation of linear momentum for particles.
- To analyze the mechanics of impact.
- To introduce the concept of angular impulse and momentum.
- To solve problems involving steady fluid streams and propulsion with variable mass.

Principle of Linear Impulse and Momentum

In this section we will integrate the equation of motion with respect to time and thereby obtain the principle of impulse and momentum. The resulting equation will be useful for solving problems involving force, velocity, and time.
Using kinematics, the equation of motion for a particle of mass can be written as

$$\Sigma F = ma = m\frac{dv}{dt} \qquad (1)$$

where $\boldsymbol{a}$ and $\boldsymbol{v}$ are both measured from an inertial frame of reference.
Rearranging the terms and integrating between the limits $v = v_1$ at $t = t_1$ and $v = v_2$ at $t = t_2$, we have

$$\sum \int_{t_1}^{t_2} F dt = m \int_{v_1}^{v_2} dv$$

Or

$$\sum \int_{t_1}^{t_2} F dt = mv_2 - mv_1 = m(v_2 - v_1) \qquad (2)$$

This equation is referred to as the principle of linear impulse and momentum. From the derivation it can be seen that it is simply a time integration of the equation of motion. It provides a direct means of obtaining the particle's final velocity v_2 after a specified time period when the particle's initial velocity is known and the forces acting on the particle are either constant or can be expressed as functions of time. By comparison, if v_2 was determined using the equation of motion, a twostep process would be necessary; i.e., apply $\sum \boldsymbol{F} = \boldsymbol{ma}$ to obtain a, then integrate $a = dv/dt$ to obtain v_2.

Linear Momentum. Each of the two vectors of the form $\boldsymbol{L} = \boldsymbol{mv}$ in Eq. (2) is referred to as the particle's linear momentum. Since $\boldsymbol{m}$ is a positive scalar, the linear-momentum vector has the same direction as $\boldsymbol{v}$, and its magnitude $\boldsymbol{mv}$ has units of mass-velocity, e.g., ***kg.m/s***, or slug ' ***ft/s***.

Linear Impulse. The integral $\boldsymbol{I} = \int \boldsymbol{F}dt$ in Eq. (2) is referred to as the linear impulse. This term is a vector quantity which measures the effect of a force during the time the force acts. Since time is a positive scalar, the impulse acts in the same direction as the force, and its magnitude has units of force-time, e.g., N.s or lb·s. *
If the force is expressed as a function of time, the impulse can be determined by direct evaluation of the integral. In particular, if the force is constant in both magnitude and direction, the resulting impulse becomes.

$$I = \int_{t_1}^{t_2} F_c dt = F_c(t_2 - t_1)$$

Graphically the magnitude of the impulse can be represented by the shaded area under the curve of force versus time, Fig. l. A constant force creates the shaded rectangular area shown in Fig. 2.

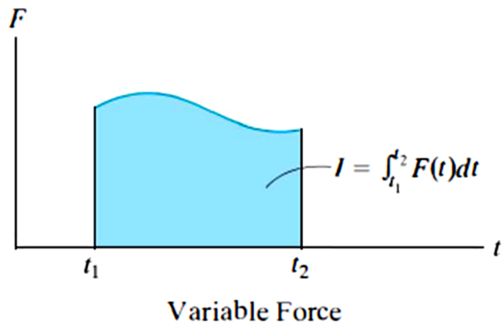


Fig. l.

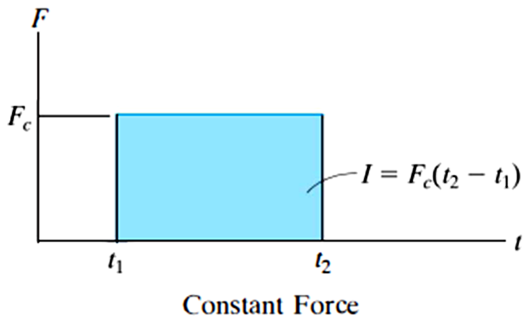


Fig. 2.

Although the units for impulse and momentum are defined differently, it can be shown that Eq. 2 is dimensionally homogeneous.

Principle of Linear Impulse and Momentum. For problem solving, Eq. 2 will be rewritten in the form

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2 \qquad (3)$$

which states that the initial momentum of the particle at time t_1 plus the sum of all the impulses applied to the particle from t_1 to t_2 is equivalent to the final momentum of the particle at time t_2. These three terms are illustrated graphically on the impulse and momentum diagrams shown in Fig. 3. The two momentum diagrams are simply outlined shapes of the particle which indicate the direction and magnitude of the particle's initial and final momenta, $\boldsymbol{mv_1}$ and $\boldsymbol{mv_2}$. Similar to the free-body diagram, the impulse diagram is an outlined shape of the particle showing all the impulses that act on the particle when it is located at some intermediate point along its path. If each of the vectors in Eq. (3) is resolved into its ***x, y, z*** components, we can write the following three scalar equations of linear impulse and momentum.

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2 \qquad (4)$$

$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z dt = m(v_z)_2$$

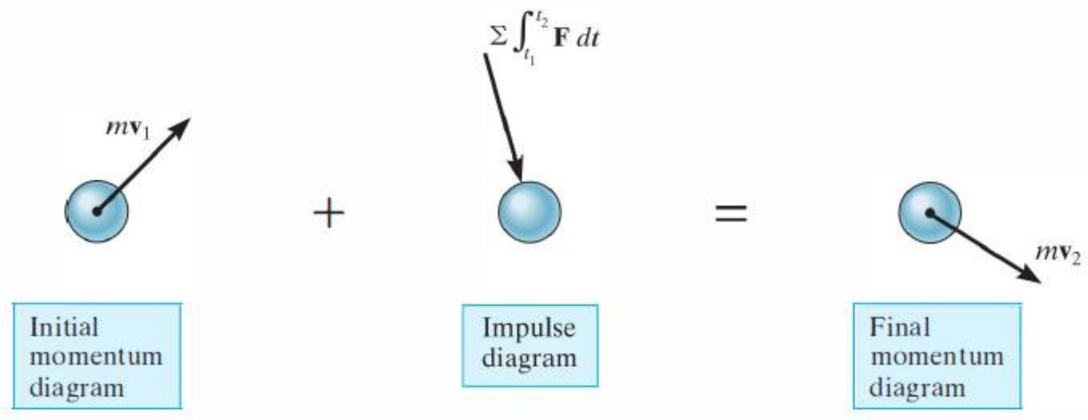


Fig. 3.

Procedure for Analysis

The principle of linear impulse and momentum is used to solve problems involving *force, time*, and *velocity*, since these terms are involved in the formulation. For application it is suggested that the following procedure be used.*

Free-Body Diagram.

- Establish the x, y, z inertial frame of reference and draw the particle's free-body diagram in order to account for all the forces that produce impulses on the particle.
- The direction and sense of the particle's initial and final velocities should be established.
- If a vector is unknown, assume that the sense of its components is in the direction of the positive inertial coordinate(s).
- As an alternative procedure, draw the impulse and momentum diagrams for the particle as discussed in reference to Fig. 15–3.

Principle of Impulse and Momentum.

- In accordance with the established coordinate system, apply the principle of linear impulse and momentum, $m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$. If motion occurs in the x–y plane, the two scalar component equations can be formulated by either resolving the vector components of $\mathbf{F}$ from the free-body diagram, or by using the data on the impulse and momentum diagrams.
- Realize that every force acting on the particle's free-body diagram will create an impulse, even though some of these forces will do no work.
- Forces that are functions of time must be integrated to obtain the impulse. Graphically, the impulse is equal to the area under the force–time curve.

EXAMPLE 15.1

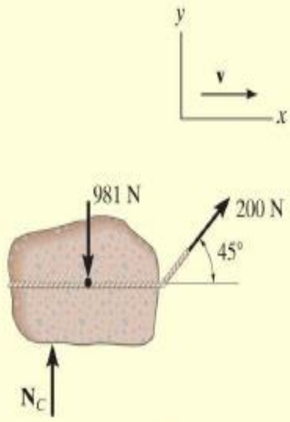
The 100-kg stone shown in Fig. 15–4*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during this time interval.

(a)

SOLUTION

This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Free-Body Diagram. See Fig. 15–4*b*. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s $[\mathbf{I} = \mathbf{F}_c(t_2 - t_1)]$. Note the alternative procedure of drawing the stone's impulse and momentum diagrams, Fig. 15–4*c*.



(b)

Principle of Impulse and Momentum. Applying Eqs. 15–4 yields

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 200\text{ N}\cos 45°(10\text{ s}) = (100\text{ kg})v_2$$

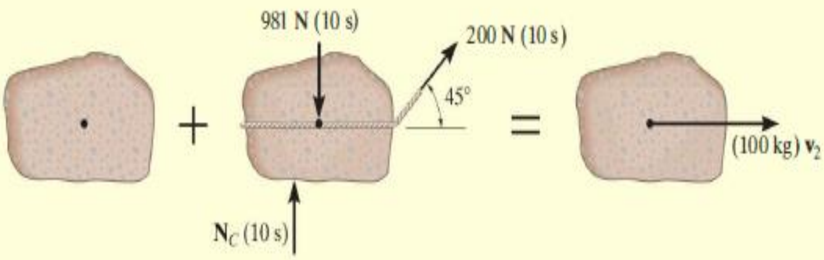
$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N}\sin 45°(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

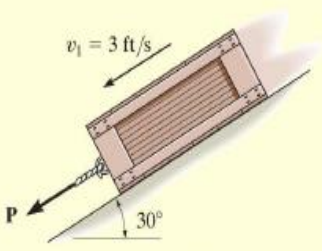
NOTE: Since no motion occurs in the *y* direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for N_C.



(c)

Fig. 15–4

EXAMPLE 15.2



(a)

The 50-lb crate shown in Fig. 15–5*a* is acted upon by a force having a variable magnitude $P = (20t)$ lb, where t is in seconds. Determine the crate's velocity 2 s after **P** has been applied. The initial velocity is $v_1 = 3$ ft/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.

SOLUTION

Free-Body Diagram. See Fig. 15–5*b*. Since the magnitude of force $P = 20t$ varies with time, the impulse it creates must be determined by integrating over the 2-s time interval.

Principle of Impulse and Momentum. Applying Eqs. 15–4 in the x direction, we have

$$(+\swarrow) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$\frac{50\text{ lb}}{32.2\text{ ft/s}^2}(3\text{ ft/s}) + \int_0^{2\text{ s}} 20t\,dt - 0.3N_C(2\text{ s}) + (50\text{ lb})\sin 30^\circ(2\text{ s}) = \frac{50\text{ lb}}{32.2\text{ ft/s}^2}v_2$$

$$4.658 + 40 - 0.6N_C + 50 = 1.553v_2$$

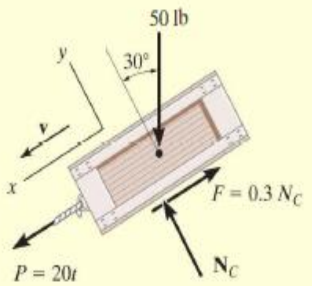
The equation of equilibrium can be applied in the y direction. Why?

$$+\nwarrow\Sigma F_y = 0; \qquad N_C - 50\cos 30^\circ\text{ lb} = 0$$

Solving,

$$N_C = 43.30\text{ lb}$$

$$v_2 = 44.2\text{ ft/s}\swarrow \qquad \textit{Ans.}$$



(b)

Fig. 15–5

NOTE: We can also solve this problem using the equation of motion. From Fig. 15–5*b*,

$$+\swarrow\Sigma F_x = ma_x; \quad 20t - 0.3(43.30) + 50\sin 30^\circ = \frac{50}{32.2}a$$

$$a = 12.88t + 7.734$$

Using kinematics

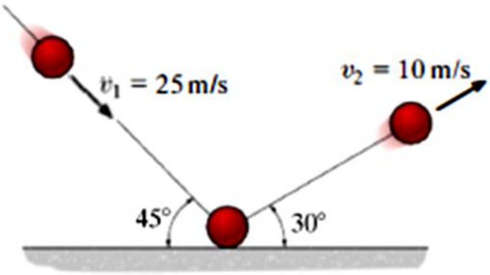
$$+\swarrow dv = a\,dt; \qquad \int_{3\text{ ft/s}}^{v} dv = \int_0^{2\text{ s}}(12.88t + 7.734)dt$$

$$v = 44.2\text{ ft/s} \qquad \textit{Ans.}$$

By comparison, application of the principle of impulse and momentum eliminates the need for using kinematics ($a = dv/dt$) and thereby yields an easier method for solution.

Example:3.

The ***0.5-kg*** ball strikes the rough ground and rebounds with the velocities shown. Determine the magnitude of the impulse the ground exerts on the ball. Assume that the ball does not slip when it strikes the ground, and neglect the size of the ball and the impulse produced by the weight of the ball.



Solution:

Free-Body Diagram. See Fig. 4.

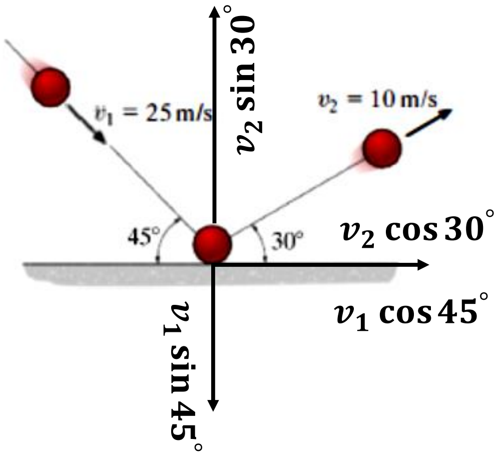


Fig. 4.

Principle of Impulse and Momentum. Applying Eqs. (4) in the x-direction, we have

At x-direction:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$I_x = \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2 - m(v_x)_1 = m[(v_x)_2 - (v_x)_1]$$

$$\therefore I_x = m[(v_x)_2 - (v_x)_1] = m[v_2 \cos 30^\circ - v_1 \cos 45^\circ] =$$

$\therefore I_x = 0.5[10\cos 30^\circ - 25\cos 45^\circ] = -4.51\ N.s$

At y-direction:

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$I_y = \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2 - m(v_y)_1 = m\left[(v_y)_2 - (v_y)_1\right]$$

$$\therefore I_y = m\left[(v_y)_2 - (v_y)_1\right] = m[v_2 \sin 30^\circ - v_1 \sin 45^\circ] =$$

$$\therefore I_y = 0.5[10\sin 30^\circ - (-25\sin 45^\circ)] = 11.339\ N.s$$

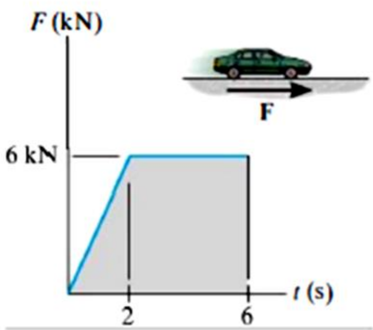
$$\therefore I = \sqrt{{I_x}^2 + {I_y}^2} = \sqrt{(-4.51)^2 + (11.339)^2} = 12.2\ N.s$$

Example4:

The wheels of the ***1.5-Mg*** car generate the traction force ***F*** described by the graph. If the car starts from rest, determine its speed when ***t = 6 s***.

Solution:

$(v_x)_1 = 0$



Principle of Impulse and Momentum. Applying Eqs. (4) in the x-direction, we have

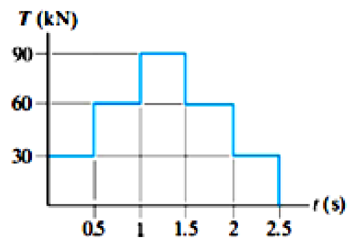
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + \frac{1}{2}(2 \times 6000) + (6-2)6000 = 1.5 \times 10^3 (v_x)_2$$

$$\therefore (v_x)_2 = \frac{\frac{1}{2}(2 \times 6000) + (6-2)6000}{1.5 \times 10^3} = 20\ m/s$$

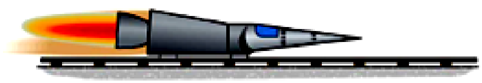
Example5:

Determine the maximum speed attained by the *1.5-Mg* rocket sled if the rockets provide the thrust shown in the graph. Initially, the sled is at rest. Neglect friction and the loss of mass due to fuel consumption.



Answer:

Data: $m = 1.5\ Mg = 1.5 \times 10^3 = 1500\ kg,\ v_1 = 0$



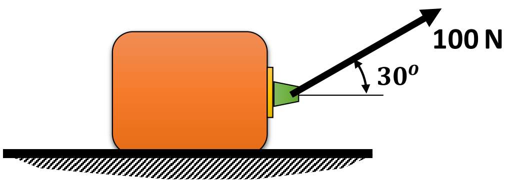
$$m(v_1)_x + \sum \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$0 + 2(30 \times 0.5) + 2(60 \times 0.5) + (90 \times 0.5) = 1500\ v_2$$

$$v_2 = \frac{2(30 \times 0.5) + 2(60 \times 0.5) + (90 \times 0.5)}{1500} = 90\ m/s$$

Example6:

If the coefficient of kinetic friction between the **150-kg** crate and the ground is $\mu_k = 0.2$, determine the speed of the crate when $t = 4\ s$. The crate starts from rest and is towed by the ***100-N*** force.



Data: m = 150 kg, P = 100 N, $\mu_k = 0.2$, and $\Delta t = 4\ s$

Solution:

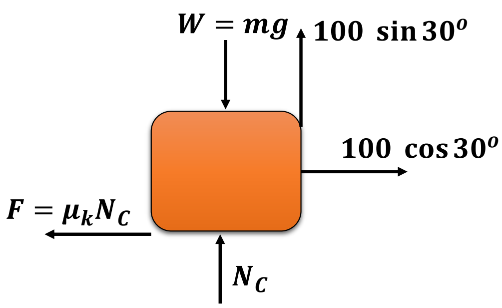
From Free-Body Diagram.

$$+\uparrow \sum F_y = ma_y = 0$$

$$100 \sin 30^o + N_C - mg = 0$$

$$\therefore N_C = (150 \times 9.81) - (100 \sin 30^o)$$

$$= 1421.5\ N$$



Free body diagram

Principle of Impulse and Momentum. Applying in the x-direction, we have

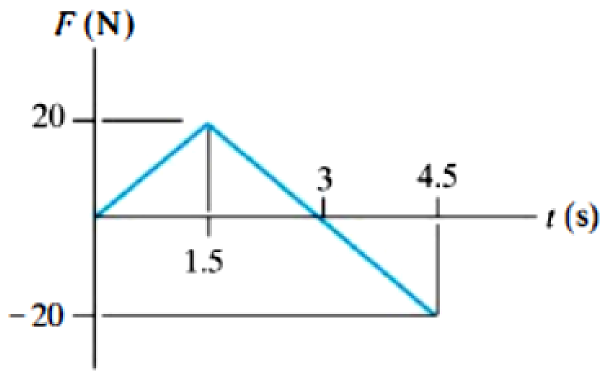
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + (100 \cos 30^o \times 4) - (0.2 \times 1421.5\ ) = 100(v_x)_2$$

$$\therefore (v_x)_2 = \frac{(100\cos 30^o \times 4) - (0.2 \times 1421.5 \times 4)}{100} = -\overrightarrow{7.91}\ m/s$$

Example7:

The ***l0-kg*** smooth block moves to the right with a velocity of $\boldsymbol{v_o = 3}$ m/s when force ***F*** is applied. If the force varies as shown in the graph, determine the velocity of the block when $\boldsymbol{t = 4.5\ s}$.



Solution:

From the graph.

By applying Principle of Impulse and Momentum

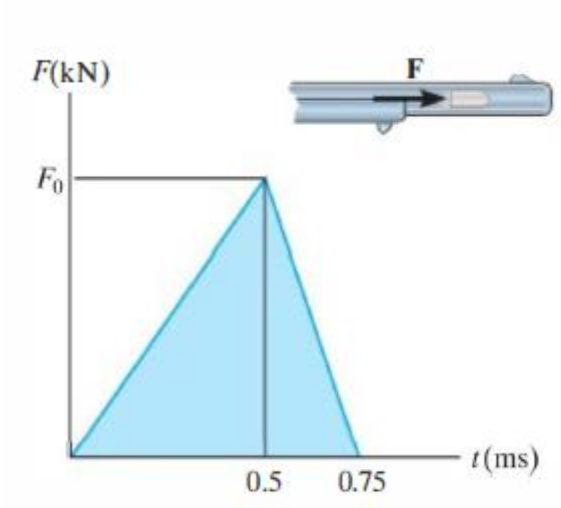
$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

$$10 \times 3 + (0.5 \times 3 \times 20) + \big(0.5 \times 1.5 \times (-20)\big) = 10v_2$$

$$\therefore v_2 = \frac{(10 \times 3) + (0.5 \times 3 \times 20) + \big(0.5 \times 1.5 \times (-20)\big)}{10} = 4.5\, m/s$$

Solved Example

1. Assuming that the force acting on a 2-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force Fo applied to the bullet when it is fired. The muzzle velocity is 500 m/s when t = 0.75 ms. Neglect friction between the bullet and the rifle barrel.



Solution:

Data: m = 2 g = 2 ×10^{-3} kg, v_1 = 0, v_2 = 500 m/s, t = 0.75 ms

1 s = 1000 ms

t = 0.75 ×10^{-3} s

Principle of Impulse and Momentum. Applying in the x-direction, we have

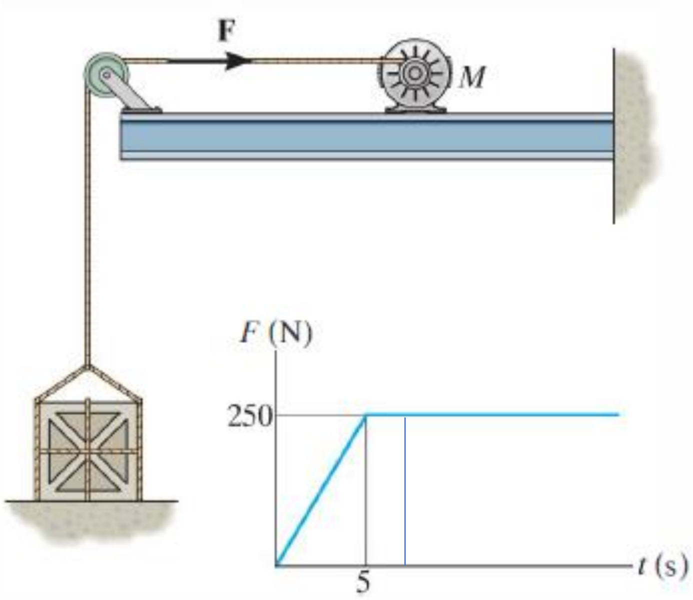
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + \frac{1}{2} F_o t = m v_2$$

$$0 + \frac{1}{2} F_o \times 0.75 = 2 \times 10^{-3} \times 500$$

$$F_o = \frac{2 \times 10^{-3} \times 500}{\frac{1}{2} \times 0.75 \times 10^{-3}} = 2666.7\ N = 2.7\ kN$$

2. The motor M pulls on the cable with a force of F, which has a magnitude that varies as shown on the graph. If the 20-kg crate is originally resting on the floor such that the cable tension is zero at the instant the motor is turned on, determine the speed of the crate when t = 6 s. Hint: First determine the time needed to begin lifting the crate.



<u>Solution:</u>

Data: m = 20 kg, t = 6 sec, $v_1 = 0$

Principle of Impulse and Momentum. Applying, we have

$$m(v)_1 + \sum \int_{t_1}^{t_2} F dt = m(v)_2$$

$$0 + \left[\frac{1}{2} 5 \times 250 + 250(6-5)\right] = 20 v_2$$

$$v_2 = \frac{\left[\frac{1}{2} 5 \times 250 + 250(6-5)\right]}{20} = 43.75\ m/s$$

Chapter (4)

Impact

Impact occurs when two bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.

In general, there are two types of impact. Central impact occurs when the direction of motion of the mass centers of the two colliding particles is along a line passing through the mass centers of the particles. This line is called the line of impact, which is perpendicular to the plane of contact, Fig. 1a. When the motion of one or both of the particles make an angle with the line of impact, Fig. 1b, the impact is said to be oblique impact.

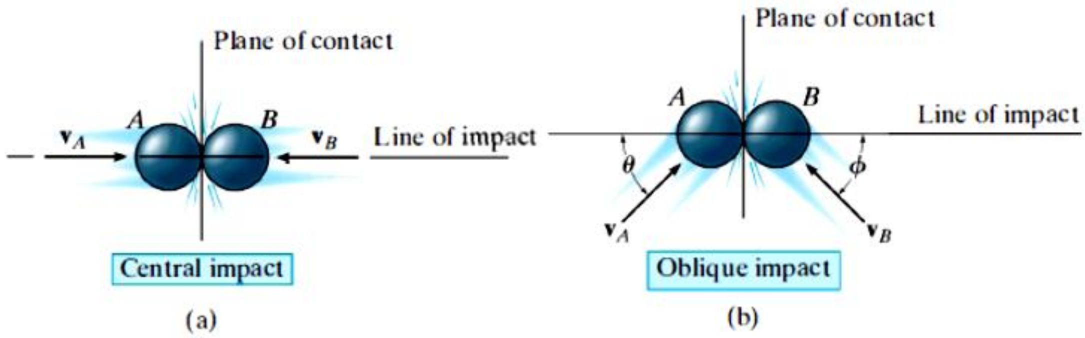


Fig. 1:

Central Impact. To illustrate the method for analyzing the mechanics of impact, consider the case involving the central impact of the two particles ***A*** and ***B*** shown in Fig. 2.
The particles have the initial momenta shown in Fig. 2a. Provided $(v_A)_1 > (v_B)_1$, collision will eventually occur.

• During the collision the particles must be thought of as deformable or nonrigid. The particles will undergo a period of deformation such that they exert an equal but opposite deformation impulse Ip dt on each other, Fig. 2b.

• Only at the instant of maximum deformation will both particles move with a common velocity v, since their relative motion is zero, Fig. 2c.

• Afterward a period of restitution occurs, in which case the particles will either return to their original shape or remain permanently deformed. The equal but opposite *restitution impulse* $\int \boldsymbol{R}\,\boldsymbol{dt}$ pushes the particles apart from one another, Fig. 2d. In reality, the physical properties of any two bodies are such that the deformation impulse with *always be greater* than that of restitution, i.e., $\int \boldsymbol{P}\,\boldsymbol{dt} > \int \boldsymbol{R}\,\boldsymbol{dt}$.

• Just after separation the particles will have the final momenta shown in Fig. 2e, where $(v_b)_2 > (v_A)_2$.

In most problems the initial velocities of the particles will be known, and it will be necessary to determine their final velocities $(v_A)_2$ and $(v_B)_2$. In this regard, momentum for the system of particles is conserved since during collision the internal impulses of deformation and restitution cancel. Hence, referring to Fig. 2a and Fig. 2e we require

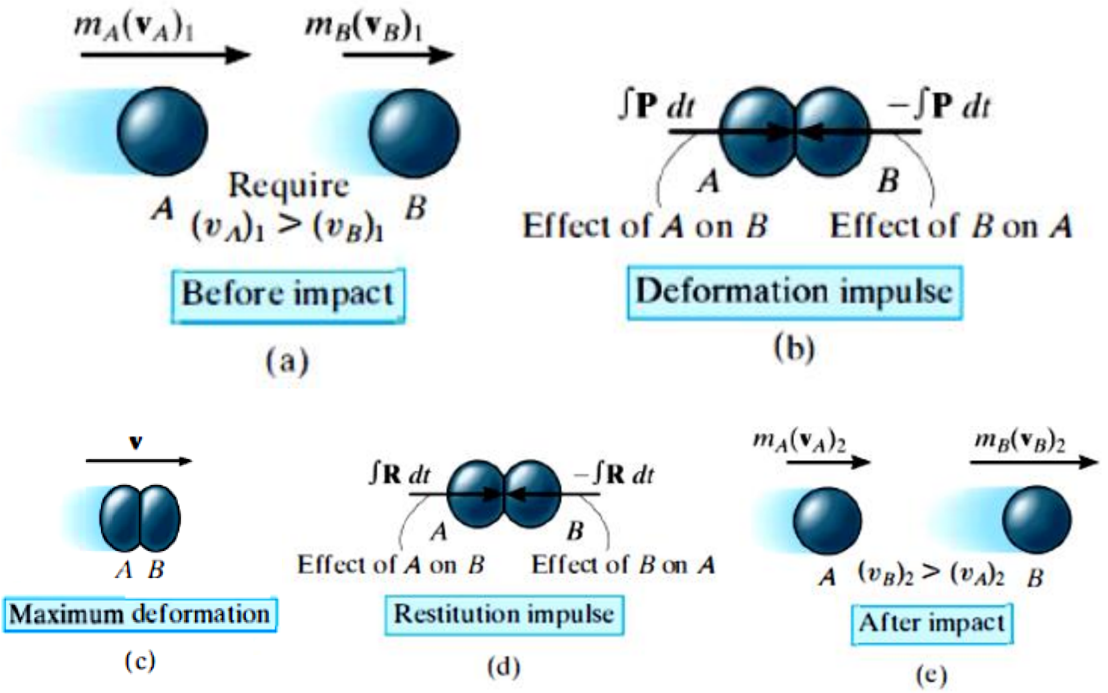


Fig. 2

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2 \qquad (1)$$

In order to obtain a second equation necessary to solve for $(\mathbf{v}_A)_2$ and $(\mathbf{v}_B)_2$, we must apply the principle of impulse and momentum to each particle. For example, during the deformation phase for particle ***A***, Figs. 2a, 2b, and 2c, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 - \int P\, dt = m_A v$$

For the restitution phase, Figs. 2c, 2d, and 2e,

$$(\overset{+}{\rightarrow}) \qquad m_A v - \int R\, dt = m_A(v_A)_2$$

The ratio of the restitution impulse to the deformation impulse is called the ***coefficient of restitution***, ***e***. From the above equations, this value for particle ***A*** is

$$e = \frac{\int R\, dt}{\int P\, dt} = \frac{v - (v_A)_2}{(v_A)_1 - v}$$

In a similar manner, we can establish ***e*** by considering particle ***B***, Fig. 2. This yields

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - v}{v - (v_B)_1}$$

If the unknown v is eliminated from the above two equations, the coefficient of restitution can be expressed in terms of the particles' initial and final velocities as

$$(\overset{\rightarrow}{+}) \qquad e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \qquad (2)$$

Provided a value for e is specified, Eqs. (1) and (2) can be solved simultaneously to obtain $(v_A)_2$ and $(v_B)_2$. In doing so, however, it is important to carefully establish a sign convention for defining the positive direction for both v_A and v_B and then use it consistently when writing both equations. As noted from the application shown, and indicated symbolically by the arrow in parentheses, we have defined the positive direction to the right when referring to the motions of both ***A*** and ***B***. Consequently, if a negative value results from the solution of either $(v_A)_2$ or $(v_B)_2$, it indicates motion is to the left.

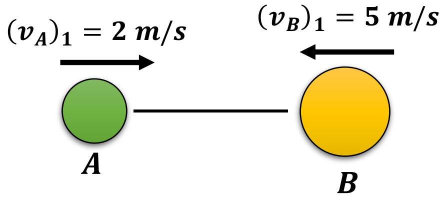
Coefficient of Restitution. From Figs. 2a and 2e, it is seen that Eq. (2) states that e is equal to the ratio of the relative velocity of the particles' separation just after impact, $(v_B)_2 - (v_A)_2$, to the relative velocity of the particles' approach just before impact, $(v_B)_1 - (v_A)_1$. By measuring these relative velocities experimentally, it has been found that e varies appreciably with impact velocity as well as with the size and shape of the colliding bodies. For these reasons the coefficient of restitution is reliable only when used with data which closely approximate the conditions which were known to exist when measurements of it were made. In general ***e*** has a value between zero and one, and one should be aware of the physical meaning of these two limits.

Elastic Impact (e = 1). If the collision between the two particles is perfectly elastic, the deformation impulse $(\int \boldsymbol{P\,dt})$. is equal and opposite to the restitution impulse $(\int \boldsymbol{R\,dt})$. Although in reality this can never be achieved, ***e = 1*** for an elastic collision.

Plastic Impact (e = 0). The impact is said to be inelastic or plastic when ***e = 0***. In this case there is no restitution impulse$(\int \boldsymbol{P\,dt})$, so that after collision both particles couple or stick together and move with a common velocity.

Example 1:

Disks ***A*** and ***B*** have a mass of ***2 kg*** and ***4 kg***, respectively. If they have the velocities shown, and ***e = 0.4***, determine their velocities just after direct central impact.



Solution:

Conservation of Momentum. In reference to the momentum diagrams, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{\rightarrow}{+}) \qquad 2(2) + 4(-5) = 2(v_A)_2 + 4(v_B)_2$$

$$\therefore 2(v_A)_2 + 4(v_B)_2 = -16 \qquad (1)$$

Coefficient of Restitution.

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$\therefore 0.4 = \frac{(v_B)_2 - (v_A)_2}{2 - (-5)}$$

$$\therefore (v_B)_2 - (v_A)_2 = 2.8 \qquad (2)$$

Solving Eqs. (1) and (2) for $(v_B)_2 - (v_A)_2$ yields

$$(v_A)_2 = -4.533\,m/s = 4.533\,m/s \leftarrow$$

$$(v_B)_2 = -1.733\,m/s = 1.733\,m/s \leftarrow$$

Oblique Impact. When oblique impact occurs between two smooth particles, the particles move away from each other with velocities having unknown directions as well as unknown magnitudes. Provided the initial velocities are known, then four unknowns are present in the problem. As shown in Fig. 3a, these unknowns may be represented either as $(v_A)_2$, $(v_B)_2$, θ_2, and ϕ_2, or as the ***x*** and ***y*** components of the final velocities.

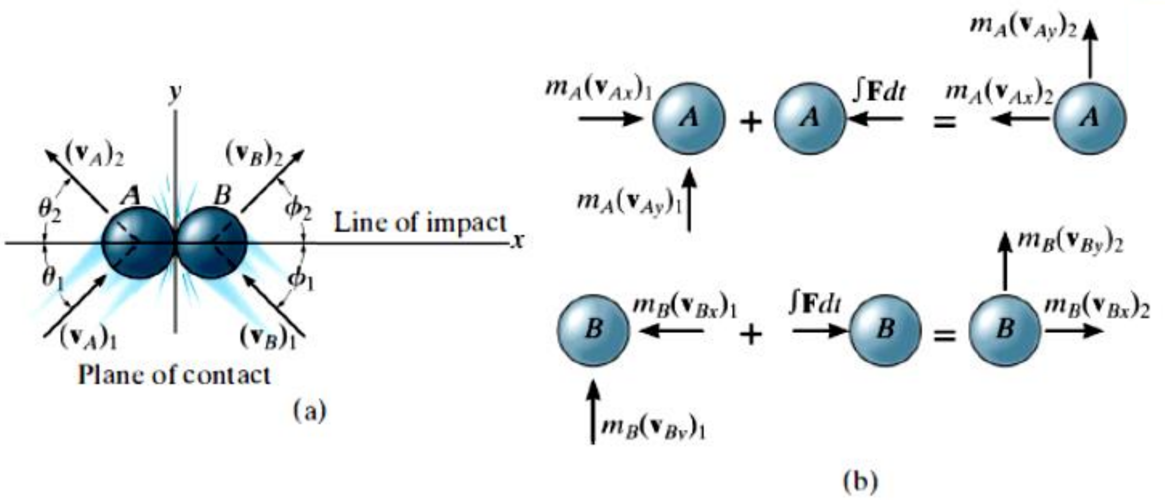


Fig. 3:

Procedure for Analysis (Oblique Impact)

If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act only in the ***x direction***, Fig. 3b. By resolving the velocity or momentum vectors into components along the ***x*** and ***y axes***, Fig. 3b, it is then possible to write four independent scalar equations in order to determine $(v_{Ax})_2$, $(v_{Ay})_2$, $(v_{Bx})_2$, and $(v_{By})_2$.

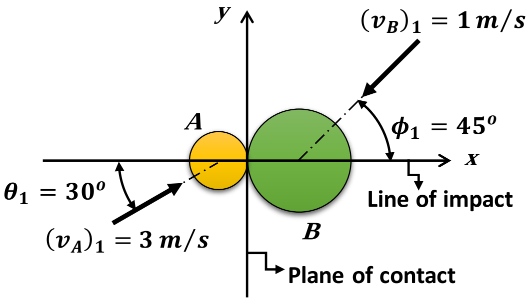
· Momentum of the system is conserved along the line of impact, ***x axis***, so that $\sum m(v_x)_1 = \sum m(v_x)_2$

· The coefficient of restitution, $e = [(v_{Bx})_2 - (v_{Ax})_2 / [(v_{Ax})_1 - (v_{Bx})_1]$, relates the relative velocity components of the particles along the line of impact (***x axis***).

· If these two equations are solved simultaneously, we obtain $(v_{Ax})_2$ and $(v_{Bx})_2$.
· Momentum of particle ***A*** is conserved along the ***y axis***, perpendicular to the line of impact, since no impulse acts on particle ***A*** in this direction. As a result $m_A(v_{Ay})_1 = m_A(v_{Ay})_2$ or $(v_{Ay})_1 = (v_{Ay})_2$
· Momentum of particle B is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle B in this direction. Consequently $(v_{By})_1 = (v_{By})_2$.
Application of these four equations is illustrated in **Example 2**.

Example 2:

Two smooth disks A and E, having a mass of 1 kg and 2 kg, respectively, collide with the velocities shown in Figure. If the coefficient of restitution for the disks is e = 0.75, determine the x and y components of the final velocity of each disk just after collision.



Solution:

Resolving each of the initial velocities into x and y components, we have

$(v_{Ax})_1 = 3\cos 30^o = 2.598\ m/s$ $\quad$ $(v_{Ay})_1 = 3\sin 30^o = 1.50\ m/s$

$(v_{Bx})_1 = -1\cos 45^o = -0.7071\ m/s$ $\quad$ $(v_{By})_1 = -1\sin 45^o = -0.7071\ m/s$

Since the impact occurs in the x direction (line of impact), the conservation of momentum for both disks can be applied in this direction.

Conservation of "x" Momentum. In reference to the momentum diagrams, we have

$$m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$1(2.598) + 2(-0.707) = 1(v_{Ax})_2 + 2(v_{Bx})_2$$

$$\therefore (v_{Ax})_2 + 2(v_{Bx})_2 = 1.184 \qquad (1)$$

Coefficient of Restitution (x).

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

$$0.75 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{2.598 - (-0.7071)}$$

$\therefore (v_{Bx})_2 - (v_{Ax})_2 = 2.479$ (2)

Solving Eqs. (1) and (2) for $(v_{Bx})_2 - (v_{Ax})_2$ yields

$(v_{Ax})_2 = -1.26\ m/s \leftarrow$ and $(v_{Ax})_2 = 1.22\ m/s \rightarrow$

Conservation of "y" Momentum. The momentum of each disk is conserved in the y direction (plane of contact) since the disks are smooth and therefore no external impulse acts in this direction.

$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \therefore (v_{Ay})_2 = 1.50\ m/s \uparrow$

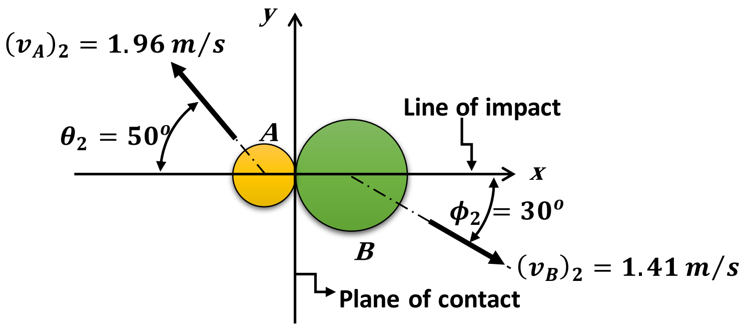
$m_B(v_{By})_1 = m_B(v_{By})_2 \quad \therefore (v_{By})_2 = -0.707\ m/s \uparrow = 0.707\ m/s \downarrow$

$(v_A)_2 = \sqrt{(-1.26)^2 + (1.50)^2} = 1.96\ m/s$

$(v_B)_2 = \sqrt{(1.22)^2 + (0.707)^2} = 1.41\ m/s$

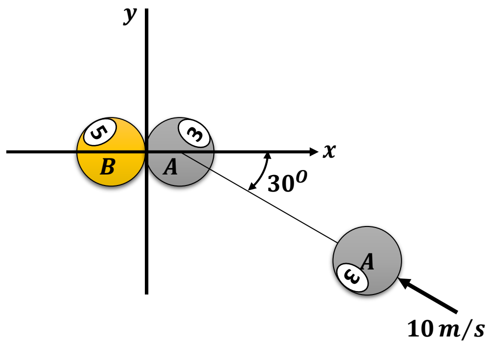
$\therefore \theta_2 = \tan^{-1}\frac{(v_{Ay})_2}{(v_{Ax})_2} = \tan^{-1}\frac{1.50}{-1.26} = -50^o \quad \therefore \theta_2 = 180 - 50 = 130^o$ from x-axis

$\therefore \phi_2 = \tan^{-1}\frac{(v_{By})_2}{(v_{Bx})_2} = \tan^{-1}\frac{-0.707}{1.22} = -30^o \quad \therefore \phi_2 = 360 - 30 = 330^o$ from x-axis



Example 3:

The pool ball A travels with a velocity of 10 m/s just before it strikes ball B, which is at rest. If the masses of A and B are each 200 g, and the coefficient of restitution between them is e = 0.8, determine the velocity of both balls just after impact.

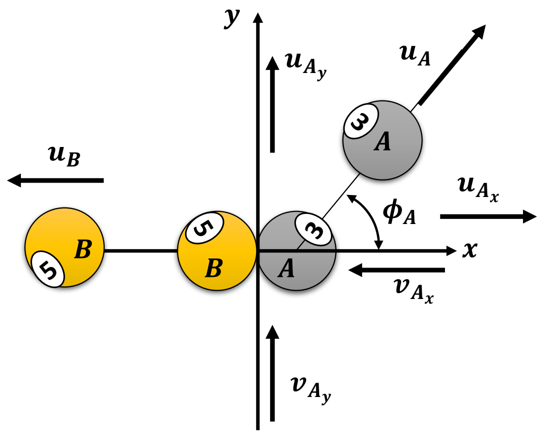


Solution:

Resolving each of the initial velocities into x and y components, we have

$v_{Ax} = 10\cos 30^o = -8.66\ m/s$ $\quad$ $v_{Ay} = 10\sin 30^o = 5\ m/s \uparrow$

$v_{Bx} = 0 \qquad v_{By} = 0$



Since the impact occurs in the x direction (line of impact), the conservation of momentum for both disks can be applied in this direction.

Conservation of "x" Momentum. In reference to the momentum diagrams, we have

$$m_A v_{Ax} + m_B v_{Bx} = m_A u_{Ax} + m_B u_{Bx}$$

$$0.2(-8.66) + 0 = 0.2u_{Ax} + 0.2u_{Bx}$$

$$\therefore u_{Ax} + u_{Bx} = -8.66 \qquad (1)$$

Coefficient of Restitution (x).

$$e = \frac{u_{Bx} - u_{Ax}}{v_{Ax} - v_{Bx}} = \frac{u_{Bx} - u_{Ax}}{-8.66 - 0} = 0.8$$

$$\therefore u_{Bx} - u_{Ax} = -6.928$$

$$\therefore u_{Bx} = -6.928 + u_{Ax} \qquad (2)$$

From Eqns. (1) and (2) we get

$$u_{Ax} - 6.928 + u_{Ax} = -8.66$$

$\therefore u_{Ax} = -0.866\,m/s \leftarrow$ and $u_{Bx} = -7.794\,m/s \leftarrow$

Conservation of "y" Momentum. The momentum of each disk is conserved in the y direction (plane of contact), since the disks are smooth and therefore no external impulse acts in this direction.

$m_A v_{Ay} = m_A u_{Ay} \qquad \therefore u_{Ay} = 5\ m/s \uparrow$

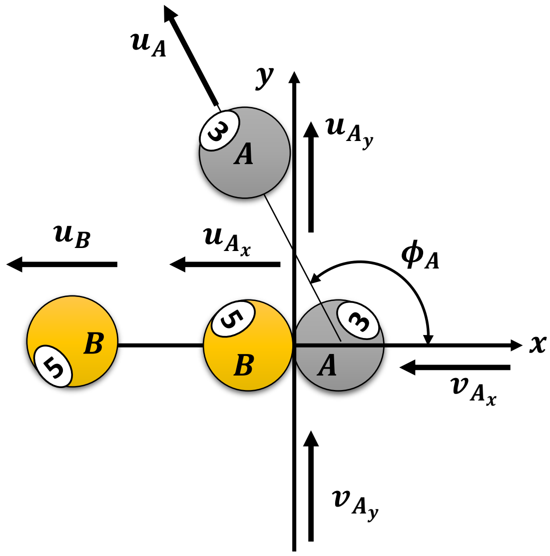
$m_B v_{By} = m_B u_{By} \quad \therefore u_{By} = 0$

$u_A = \sqrt{(-0.866)^2 + (5)^2} = 5.074\ m/s$

$u_B = \sqrt{(-7.794)^2 + (0)^2} = -7.794\ m/s$

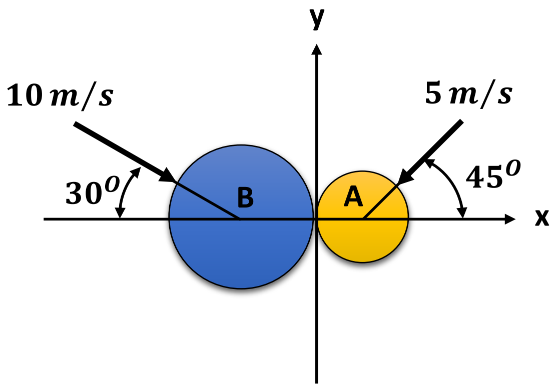
$\therefore \phi_A = \tan^{-1}\frac{u_{Ay}}{u_{Ax}} = \tan^{-1}\left(\frac{5}{-0.866}\right) = -80.2^o \quad \therefore \theta_2 = 180 - 80.2 = 99.83^o$ from x-axis

$\therefore \phi_B = \tan^{-1}\frac{u_{By}}{u_{Bx}} = \tan^{-1}\left(\frac{0}{-7.794}\right) = 0$



Example 4:

Two disks A and B weigh 2 kg and 5 kg, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their velocities just after impact. The coefficient of restitution between the disks is e = 0.6.



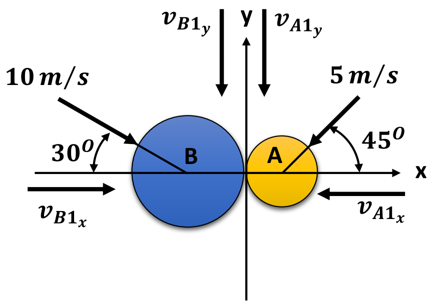
Solution:

Resolving each of the initial velocities into x and y components, we have

$(v_{Ax})_1 = -5\cos 45^o = -3.54\ m/s \qquad (v_{Ay})_1 = -5\sin 45^o = -3.54\ m/s$

$(v_{Bx})_1 = 10\cos 30^o = 8.66\ m/s \qquad (v_{By})_1 = -10\sin 30^o = -5\ m/s$

Since the impact occurs in the x direction (line of impact), the conservation of momentum for both disks can be applied in this direction.



Conservation of "x" Momentum. In reference to the momentum diagrams, we have

$m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$

$2(-3.54) + 5(8.66) = 2(v_{Ax})_2 + 5(v_{Bx})_2$

$\therefore 2(v_{Ax})_2 + 5(v_{Bx})_2 = 36.22$ (1)

Coefficient of Restitution (x).

$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$

$0.6 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{-3.54 - (8.66)}$

$\therefore (v_{Bx})_2 - (v_{Ax})_2 = -7.32$ (2)

Solving Eqs. (1) and (2) for $(v_{Bx})_2 - (v_{Ax})_2$ yields

$(v_{Ax})_2 = 10.40\ m/s \rightarrow$ and $(v_{Bx})_2 = 3.1\ m/s \rightarrow$

Conservation of "y" Momentum. The momentum of each disk is conserved in the y direction (plane of contact) since the disks are smooth and therefore no external impulse acts in this direction.

$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \therefore (v_{Ay})_2 = -3.54\ m/s \downarrow$

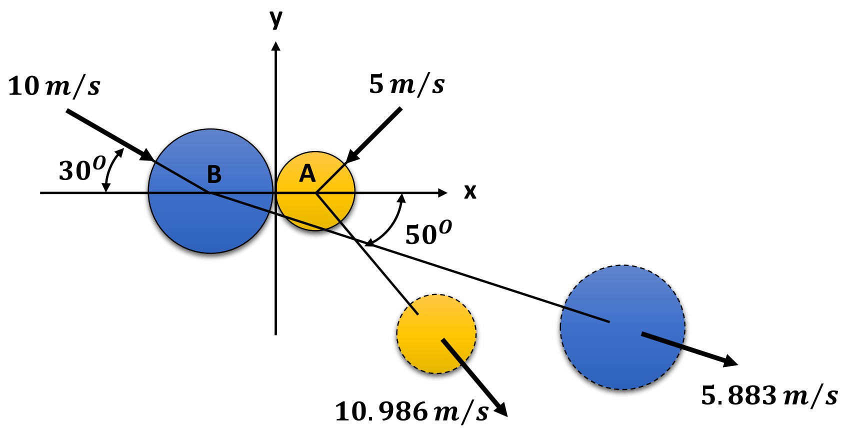
$m_B(v_{By})_1 = m_B(v_{By})_2 \quad \therefore (v_{By})_2 = -5\ m/s \downarrow$

$(v_A)_2 = \sqrt{((v_{Ax})_2)^2 + \left((v_{Ay})_2\right)^2} = \sqrt{(10.40)^2 + (-3.54)^2} = 10.986\ m/s$

$(v_B)_2 = \sqrt{((v_{Bx})_2)^2 + \left((v_{By})_2\right)^2} = \sqrt{(3.1)^2 + (-5)^2} = 5.883\ m/s$

$\therefore \theta_2 = \tan^{-1}\frac{(v_{Ay})_2}{(v_{Ax})_2} = \tan^{-1}\frac{-3.54}{10.4} = -18.8^o \quad \therefore \theta_2 = 360 - 50 = 310^o$ from x-axis

$\therefore \phi_2 = \tan^{-1}\frac{(v_{By})_2}{(v_{Bx})_2} = \tan^{-1}\frac{-5}{3.1} = -58.2^o \quad \therefore \phi_2 = 360 - 58.2 = 301.8^o$ from x-axis


y
10 m/s
5 m/s
30^o
B
A
x
50^o
10.986 m/s
5.883 m/s