Chapter (1)

Force system

1. *Analytical method*

The magnitude and direction of the resultant force may be obtained, analytically, as discussed below:

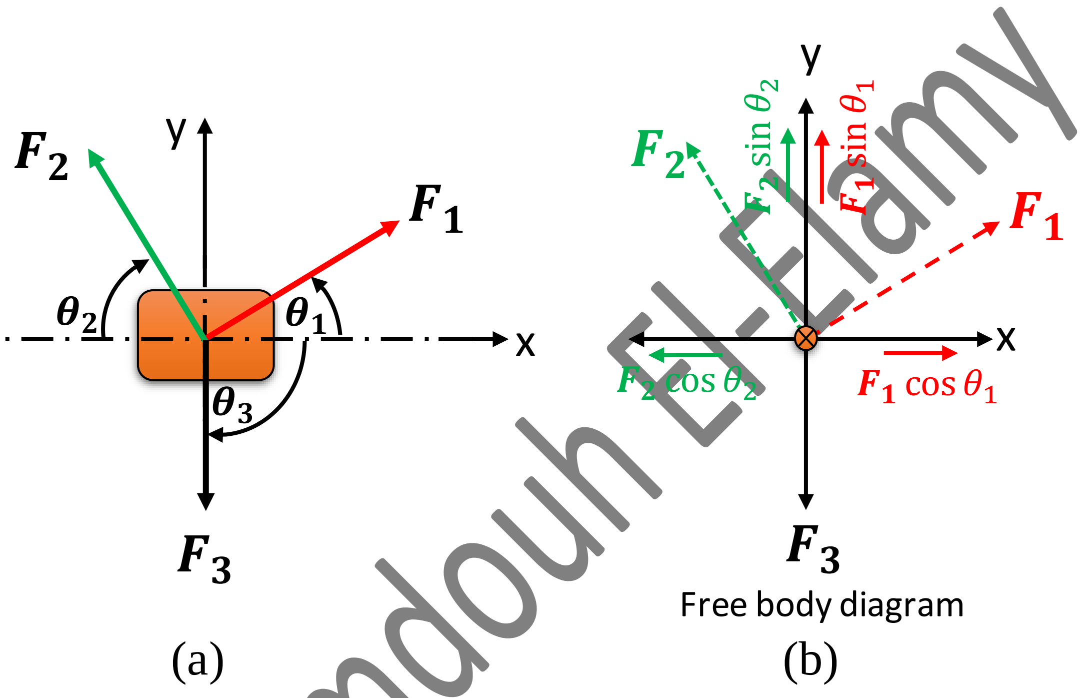


Fig. 1

1. First of all, analysis all forces into two components in free body diagram as shown in Fig. 1 (b).

2. Resolve the forces horizontally and vertically and find their sums, *i.e.* $\sum F_x$ and $\sum F_y$. We know that

Sum of horizontal components of the forces at x-axis,

$$\sum F_x = F_1 \cos\theta_1 - F_2 \cos\theta_2 \qquad (1)$$

Sum of vertically components of the forces at y-axis,

$$\sum F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 - F_3 \qquad (2)$$

3. Magnitude of the resultant force,

$$R = \sqrt{(\sum F_x)^2 + \left(\sum F_y\right)^2} \tag{3}$$

4. If θ_R is the angle, which the resultant force makes with the horizontal, then

$$\theta_R = \tan^{-1}\left(\sum F_y / \sum F_x\right) \tag{4}$$

2. *Graphical method*

The magnitude and position of the resultant force may also be obtained graphically as discussed below:

1. First of all, draw the space diagram with the positions of the several Forces, as shown in Fig. 1 (*a*).

2. Select suitable scale for all forces**.**

3. Draw the force diagram as shown in Fig.2.

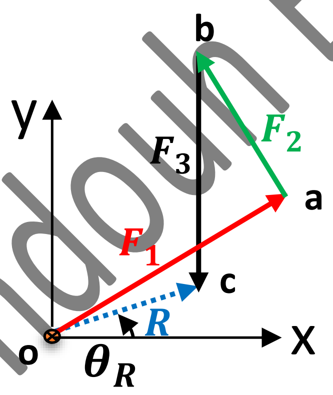


Fig. 2: Force diagram

3. Determine the resultant force by using equation (5)

$$R = \overline{oc} \times scale$$

4. The direction of resultant force can be estimated from Fig.2.

Example: 1

The screw eye in Fig. 3 is subjected to two forces, F_1 and F_2.

Determine the magnitude and direction of the resultant force.

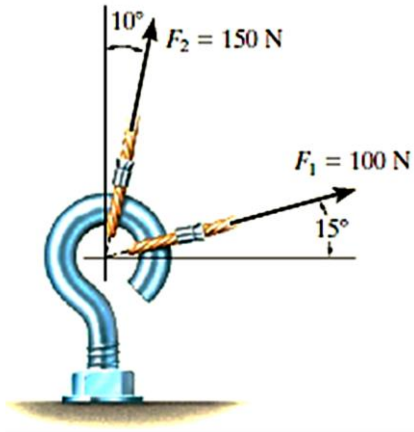
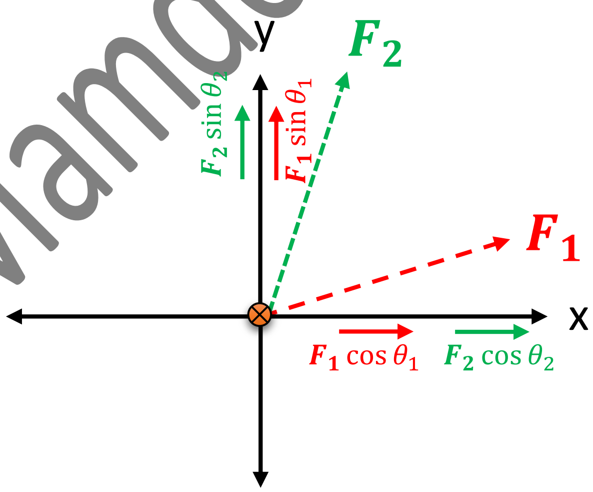


Fig. 3: Screw eye.

Solution:

1. *Analytical method*

Draw free body diagram and analysis all forces into two components in x and y directions.



Free body diagram

From free body diagram we have;

$$\sum F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 = 100 \cos 15^o + 150 \cos 80^o = 122.63$$

$$\sum F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 = 100 \sin 15^o + 150 \sin 80^o = 173.60\ N$$

The resultant force can be calculated by;

$$R = \sqrt{(\sum F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(122.63)^2 + (173.60)^2} = 213\ N$$

The direction of resultant force can be calculated by;

$$\theta_R = \tan^{-1}\frac{\sum F_y}{\sum F_x} = \tan^{-1}\frac{173.60}{122.63} = 54.8^o$$

2. Graphical method

1. First of all, draw the space diagram with the positions of the several Forces, as shown in Fig. 3.

2. Select suitable scale for all forces**.**

Take allowable scale for all 1 cm = 25 N

3. Draw the force diagram as shown in Fig.4.

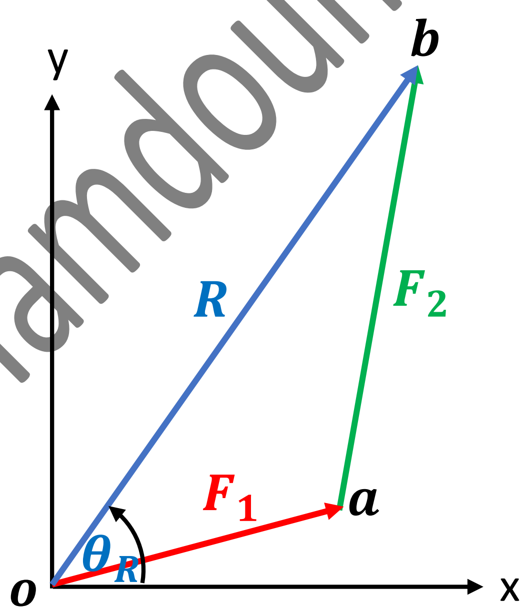


Fig. 4

From Fig. 4;

$$R = ob \times scale = 8.4 \times 25 = 210\ N$$

And

$$\theta_R = 55^o$$

Example: 2

The force F = 450 N acts on the frame shown in Fig. 5. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.

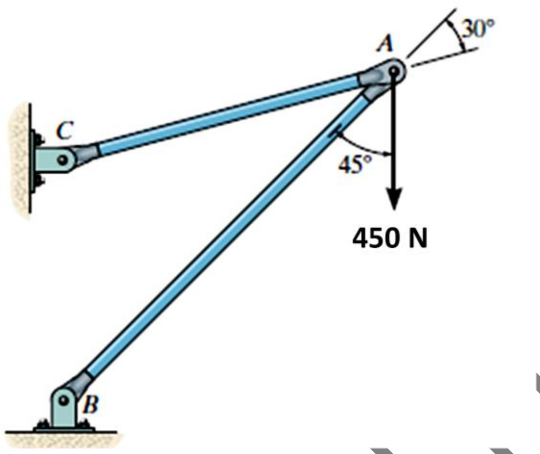
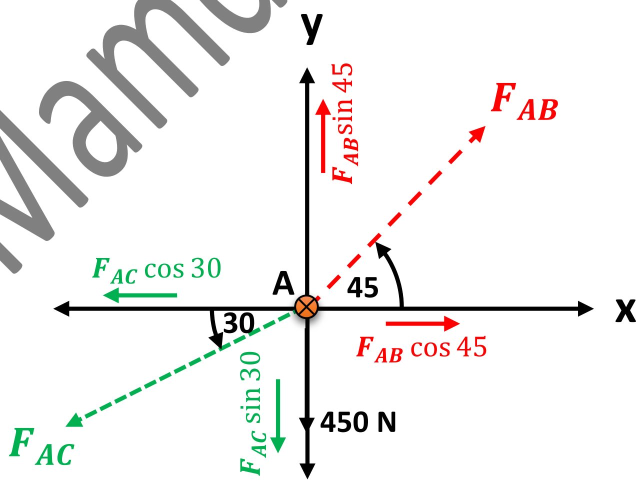


Fig. 5:

Solution:

1. *Analytical method*

Draw free body diagram and analysis all forces into two components in x and y directions.



Free body diagram

By using the Equilibrium conditions to obtain the forces $\boldsymbol{F_{AC}}$ and $\boldsymbol{F_{AB}}$.

1. $\sum F_x = 0$

$\therefore F_{AB}\cos 45 - F_{AC}\cos 30 = 0$

$$\therefore F_{AB} = F_{AC}\frac{\cos 30}{\cos 45} \qquad (1)$$

2. $\sum F_y = 0$

$$\therefore F_{AB}\sin 45 - F_{AC}\sin 30 - 450 = 0$$

$$\therefore F_{AB}\sin 45 - F_{AC}\sin 30 = 450 \qquad (2)$$

By solving Eqns. (1) and (2) we have

$$F_{AC}\frac{\cos 30}{\cos 45}\ \sin 45 - F_{AC}\sin 30 = 450$$

$$F_{AC}\frac{\sin 45}{\cos 45}\ \cos 30 - F_{AC}\sin 30 = 450$$

$$F_{AC}\tan 45\ \cos 30 - F_{AC}\sin 30 = 450$$

$$F_{AC}[\tan 45\ \cos 30 - \sin 30] = 450$$

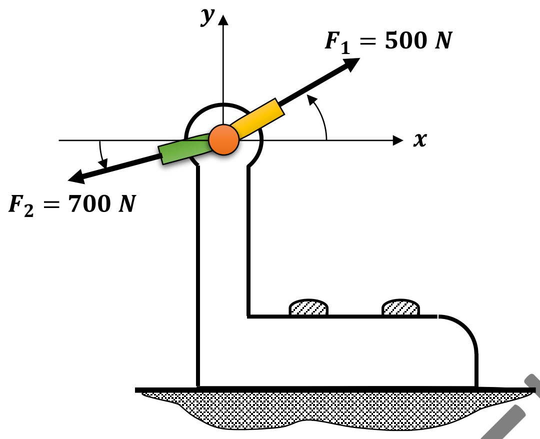
$$\therefore F_{AC} = \frac{450}{[\tan 45\ \cos 30 - \sin 30]} = 1229.4\ N$$ ***Tension force***

And

$$F_{AB} = F_{AC}\frac{\cos 30}{\cos 45} = 1229.4\frac{\cos 30}{\cos 45} = 1505.7\ N$$ ***Compression force***

Example: 3

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive *x* axis. *Using analytical and graphical methods.*



Solution:

I. Analytical Method

From the free body diagram, we get;

$\sum F_x = 500\ \cos 30^o - 700\ \cos 15^o = -243.14\ N$

$\sum F_y = 500\ \sin 30^o - 700\ \sin 15^o = +68.83\ N$

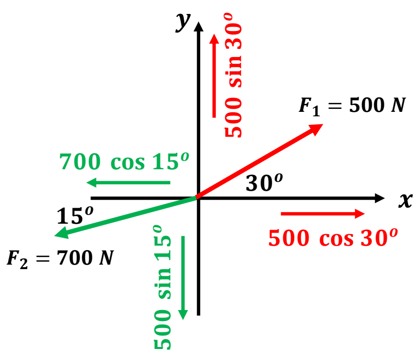
The resultant force can be given as

$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} =$
$\sqrt{(-243.14)^2 + (+68.83)^2} = 252.7\ N$

The angle of the resultant force can be given using the Equation;

$$\varphi = \tan^{-1}\frac{\sum F_y}{\sum F_x} = \tan^{-1}\frac{+68.83}{-243.14} = 15.8^o$$

$\therefore \theta = 180 - \varphi = 180^o - 15.8^o = 164.19^o$



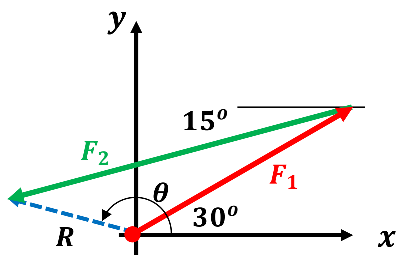
Free body diagram

II. Graphical Method

From the force polygon diagram, we get out;

The resultant force can be measured from the diagram

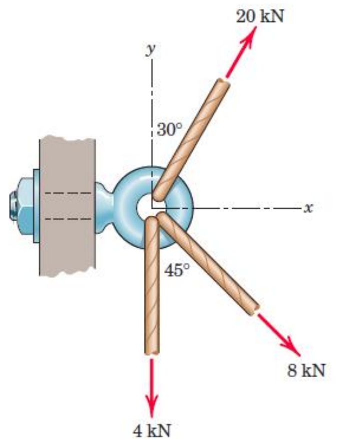
$R = 260\ N$ *and* $\theta = 165^o$



Force Polygon diagram

Example: 4

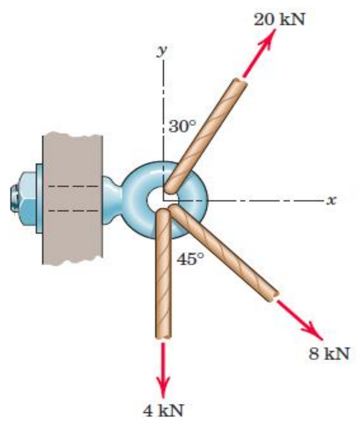
Determine the resultant R of the three tension forces acting on the eye bolt. Find the magnitude of R and the angle which R makes with the positive *x*-axis.

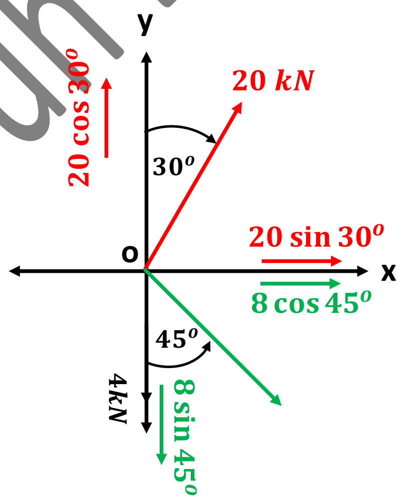


Answer:

a. Analytical Method

From the free body diagram, we have





Free body diagram

$$\sum F_x = 20 \sin 30^o + 8 \cos 45^o = 15.656\ kN$$

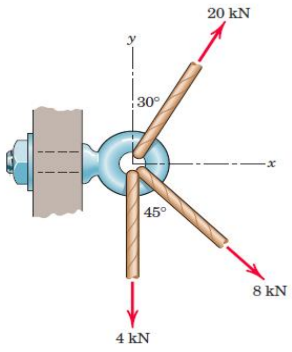
$$\sum F_y = 20 \cos 30^o - 4 - 8 \sin 45^o = 7.664\ kN$$

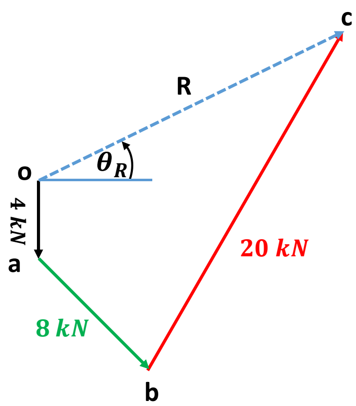
$$R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}} = \sqrt{(15.656\,)^2 + (7.664)^2} = 17.43\ kN$$

$$\theta = \tan^{-1}\frac{\sum F_y}{\sum F_x} = \tan^{-1}\frac{7.664}{15.656} = 26.08^{\circ}$$

b. Graphical Method

From force diagram we get;





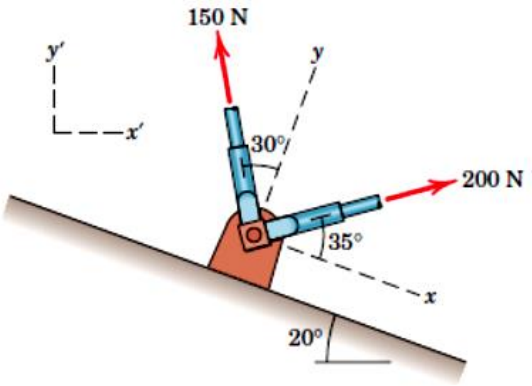
Force diagram

$\therefore R = \overline{oc} \times scale = 8.7 \times 2 = 17.4\ \ kN$

$\therefore \theta_R = 26^{\circ}$

Example: 5

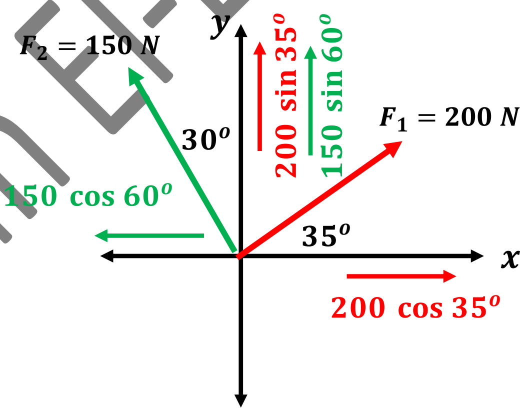
Determine the resultant **R** of the two forces applied to the bracket. Write **R** in terms of unit vectors along the *x*- and *y*-axes shown.



Solution:

I. Analytical Method

From the free body diagram, we get;



Free body diagram

$$\sum F_x = 200\ \cos 35^o - 150\ \cos 60^o$$
$$= 88.8304\ N$$

$$\sum F_y = 200\ \sin 35^o + 150\ \sin 60^o = 244.62\ N$$

The resultant force can be given as

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(88.8304)^2 + (244.62)^2} = 260.25\ N$$

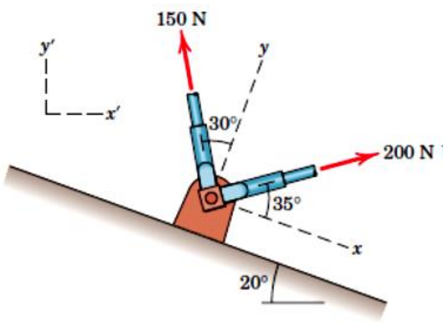
The angle of the resultant force can be given using the Equation;

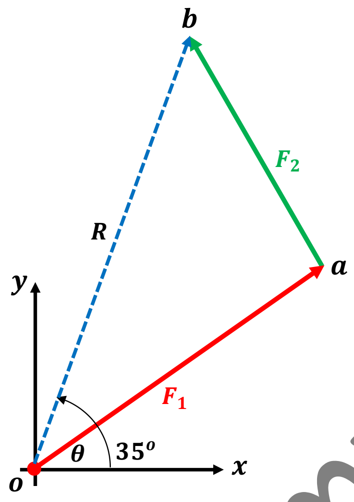
$$\varphi = \tan^{-1}\frac{\sum F_y}{\sum F_x} = \tan^{-1}\frac{244.62}{88.8304} = 70.04^o$$

$$\therefore \theta = \varphi = 70.04^o$$

II. Graphical Method

From force diagram we get;





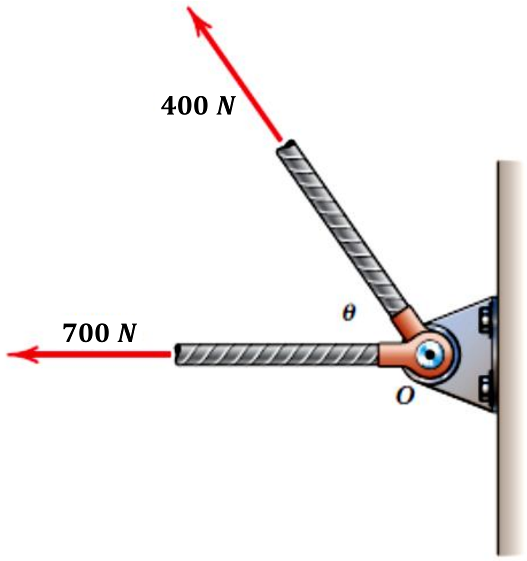
Force Polygon diagram

$\therefore R = \overline{ob} \times scale = 10.3 \times 25 = 257.5 \ kN$

$\therefore \theta_R = 70^\circ$

Example: 6

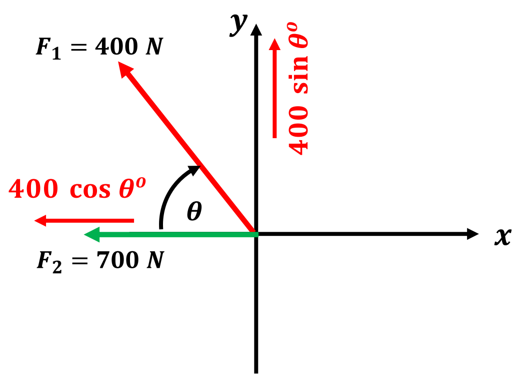
At what angle must the ***400 N*** force be applied in order that the resultant **R** of the two forces have a magnitude of ***1000 N***? For this condition what will be the angle between **R** and the horizontal?



Solution:

Analytical Method

From the free body diagram, we get;



Free body diagram

The resultant force can be given as

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\theta}$$

$$\therefore (1000)^2 = (400)^2 + (700)^2 + (2 \times 400 \times 700 \times \cos\theta)$$

$$\therefore \cos\theta = \frac{(1000)^2 - (400)^2 - (700)^2}{(2 \times 400 \times 700)}$$

$$\therefore \theta = \cos^{-1}\left[\frac{(1000)^2 - (400)^2 - (700)^2}{(2 \times 400 \times 700)}\right] = 51.31^o$$

$$\sum F_x = -400\ \cos\theta^o - 700 = -950.04\ N$$

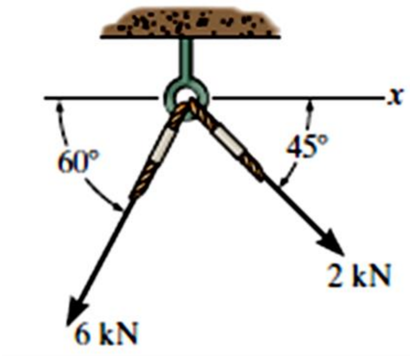
$$\sum F_y = 400\ \sin\theta^o = 312.22\ N$$

The angle of the resultant force can be given using the Equation;

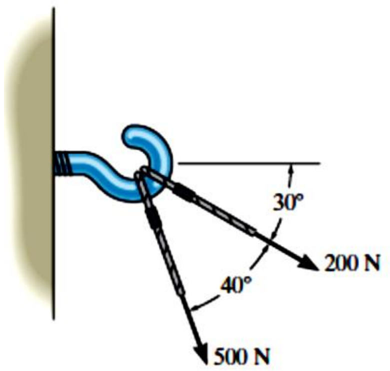
$$\beta = \tan^{-1}\frac{\sum F_y}{\sum F_x} = \tan^{-1}\frac{312.22}{-950.04} = 18.19^o$$

FUNDAMENTAL PROBLEMS

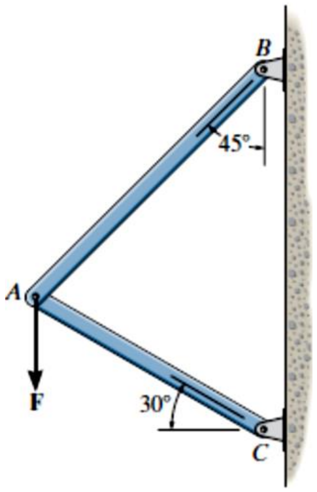
1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



2. Two forces act on the hook. Determine the magnitude of the resultant force.



3. The vertical force F acts downward at A on the two membered frame. Determine the magnitudes of the two components of F directed along the axes of AB and AC. Set F = 500 N.



4. Solve Prob. 3 with F = 350 N.

Chapter (2)

Moment of a force

Objective

- To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions.
- To provide a method for finding the moment of a force about a specified axis.

Introduction

When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called a ***torque***, but most often it is called the moment of a force or simply the ***moment***. For example, consider a wrench used to unscrew the bolt in Fig. 6 *(a)* . If a force is applied to the handle of the wrench it will tend to turn the bolt about point *O* (or the *z* axis). The magnitude of the moment is directly proportional to the magnitude of **F** and the perpendicular distance or *moment arm d*. The larger the force or the longer the moment arm, the greater the moment or turning effect. Note that if the force **F** is applied at an angle $\theta \neq 90°$, Fig. 6 *(b)* , then it will be more difficult to turn the bolt since the moment arm $d' = d \sin\theta$ will be smaller than d . If **F** is applied along the wrench, Fig. 6 *(c)*, its moment arm will be zero since the line of action of **F** will intersect point *O* (the *z* axis). As a result, the moment of **F** about *O* is also zero and no turning can occur.

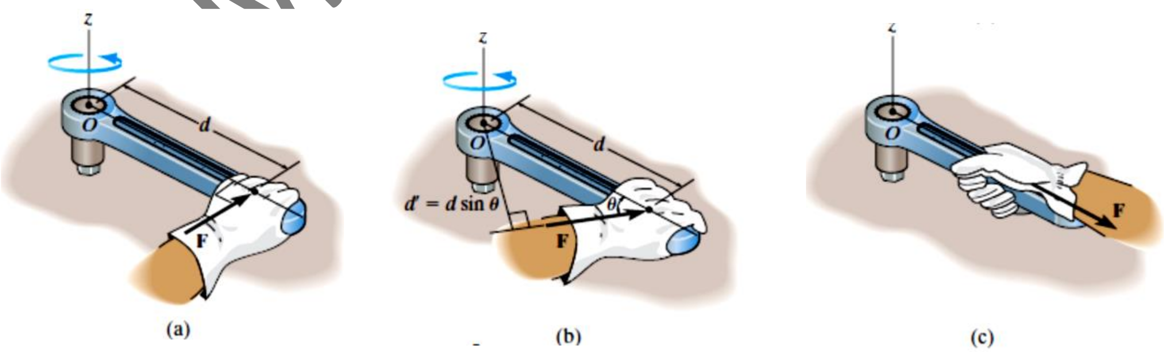


Fig. 6:

We can generalize the above discussion and consider the force **F** and point *O* which lie in the shaded plane as shown in Fig. 7 *(a)*. The moment $\boldsymbol{M_O}$ about point *O*, or about an axis passing through *O* and perpendicular to the plane, is a *vector quantity* since it has a specified magnitude and direction.

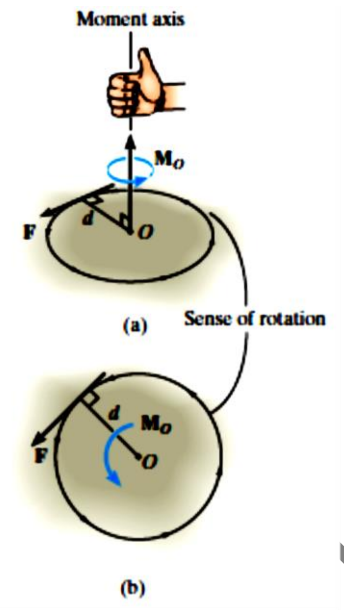


Fig. 7:

Magnitude. The magnitude of M_O is

$$M_O = F \times d \tag{3}$$

Where ***d*** is the *moment arm* or *perpendicular distance* from the axis at point ***O*** to the line of action of the force. Units of moment magnitude consist of force times distance, e.g., ***N.m*** or ***lb. ft***.

Direction. The direction of M_O is defined by its *moment axis*, which is perpendicular to the plane that contains the force **F** and its moment arm ***d***. The right-hand rule is used to establish the sense of direction of M_O. According to this rule, the natural curl of the fingers of the right hand, as they are drawn towards the palm, represent the rotation, or if no movement is possible, there is a tendency for rotation caused by the moment. As this action is performed, the thumb of the right hand will give the directional sense of M_O, Fig. 7 *(a)*. Notice that the moment vector is represented three-dimensionally by a curl around an arrow. In two dimensions this vector is represented only by the curl as in Fig. 7 *(b)*. Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page.

Resultant Moment. For two-dimensional problems, where all the forces lie within the ***x–y*** plane, Fig. 8, the resultant moment ($\mathbf{M}R$)*O* about point ***O*** (the ***z*** axis) can be determined by *finding the algebraic sum* of the moments caused by all the forces in the system. As a convention, we will generally consider ***positive moments* as *counterclockwise*** since they are directed along the positive z axis (out of the page). *Clockwise moments* will be *negative*. Doing this, the directional sense of each moment can be represented by a *plus or minus* sign. Using this sign convention, the resultant moment in Fig. 8 is therefo

$$\circlearrowleft +(M_R)_O = \sum Fd; \qquad (M_R)_O = F_1 d_1 - F_2 d_2 + F_3 d_3$$

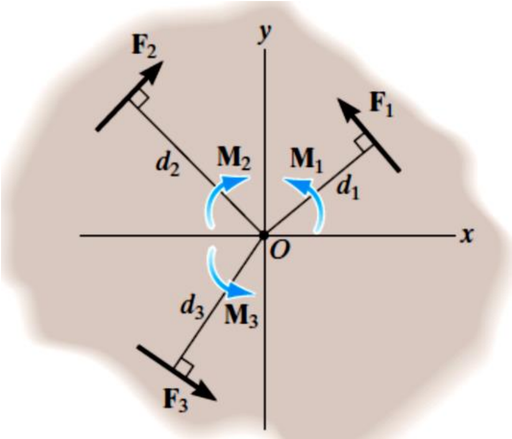


Fig. 8:

If the numerical result of this sum is a positive scalar, $(M_R)_O$ will be a counterclockwise moment (out of the page); and if the result is negative, $(M_R)_O$ will be a clockwise moment (into the page).

Example: 1

For each case illustrated in Fig. 9, determine the moment of the force about point O.

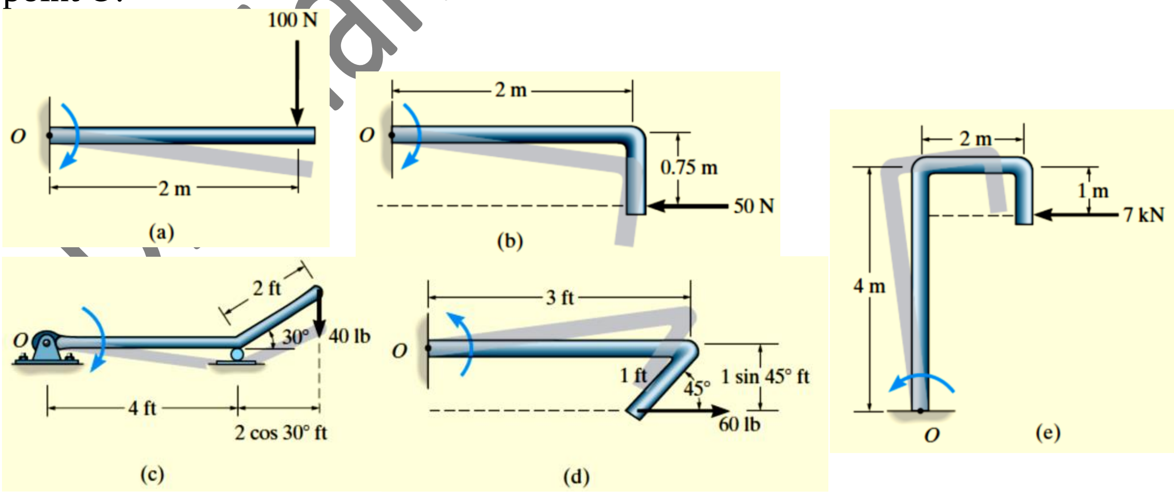


Fig. 9

Solution:

The line of action of each force is extended as a dashed line in order to establish the moment arm ***d***. Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about ***O*** is shown as a colored curl. Thus,

Fig. 9 (a) $M_O = (100\ N)(2\ m) = 200\ N.m$

Fig. 9 (b) $M_O = (50\ N)(0.75\ m) = 37.5\ N.m$

Fig. 9 (c) $M_O = (40\ Ib)(4\ ft + 2\cos 30^o\ ft) = 229\ Ib.ft$

Fig. 9 (d) $M_O = (60\ Ib)(1\sin 45^o\ \ ft) = 42.4\ Ib.ft$

Fig. 9 (b) $M_O = (7\ kN)(4\ m - 1m) = 21.0\ kN.m$

Example: 2

Determine the resultant moment of the four forces acting on the rod shown in Fig. 10 about point ***O***.

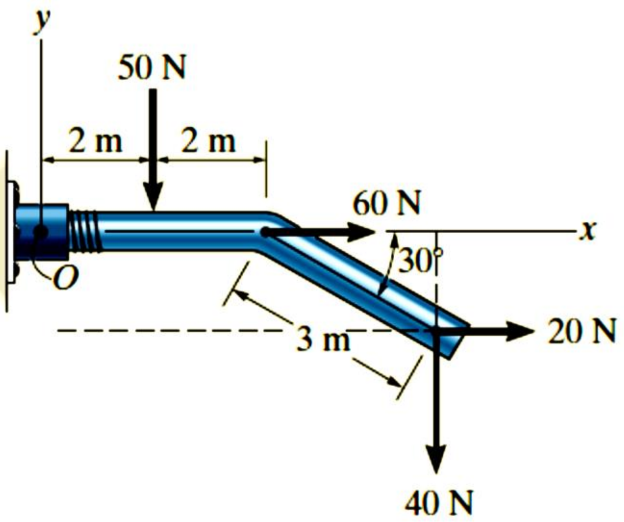


Fig. 10

Solution:

Assuming that positive moments act in the +**k** direction, i.e., counterclockwise, we have

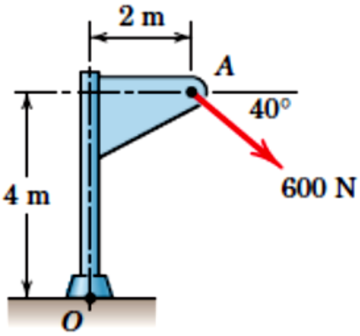
$$+(M_R)_O = \sum Fd;$$

$$(M_R)_O = -50\ N(2\ m) + 60\ N\ (0) + 20\ N\ (3\ \sin 30^o\ m) - 40\ N\ (4\ m + 3\ \cos 30^o\ m)$$

$$\therefore (M_R)_O = -334\ N.m = 334\ N.m$$

For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.

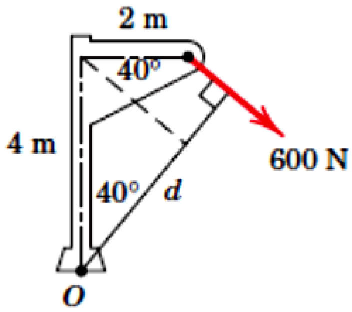
Example: 3



Calculate the magnitude of the moment about the base point O of the 600-N
force in five different ways.

Solution:

(I) The moment arm to the 600-N force is

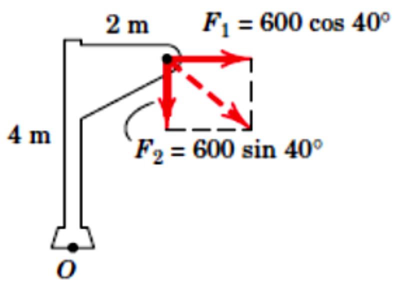


$$d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35\ \text{m}$$

(1) By $M = Fd$ the moment is clockwise and has the magnitude

$M_O = 600(4.35) = 2610$ N .m ***Ans.***

(II) Replace the force by its rectangular components at A,



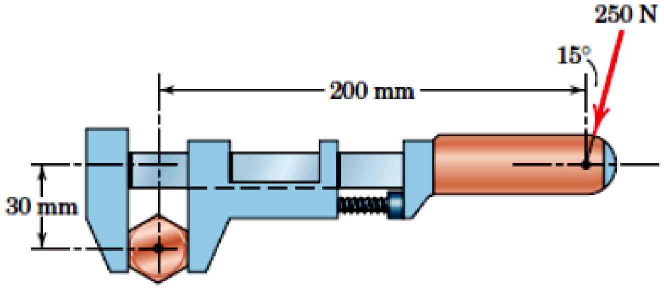
$F_1 = 600\cos 40^\circ = 460$ N, $F_2 = 600\sin 40^\circ = 386$ N

By Avignon's theorem, the moment becomes

(2) $M_O = 460(4) + 386(2) = 2610$ N .m ***Ans.***

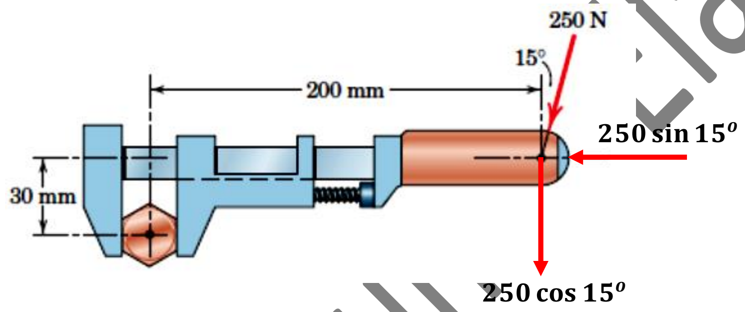
Example: 4

Calculate the moment of the ***250-N*** force on the handle of the monkey wrench about the center of the bolt.



Solution:

The moment about the center of the bolt.

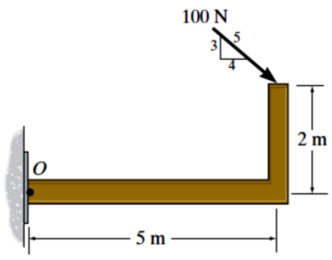


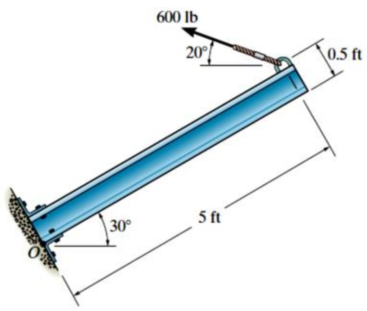
$$M = (250 \sin 15^o \times 30) - (250 \cos 15^o \times 200) = -46867.66\ N.m$$

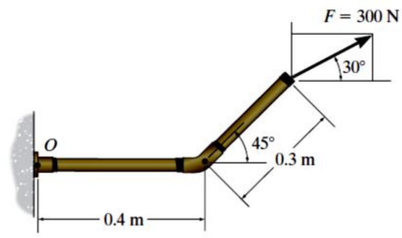
$$\therefore M = 46867.66\ N.m$$

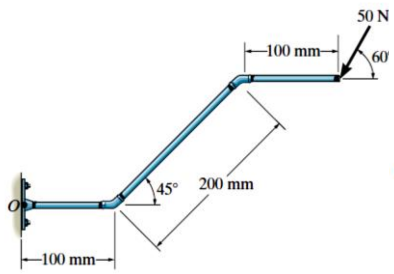
FUNDAMENTAL PROBLEMS

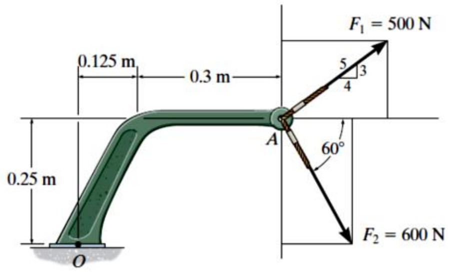
1. Determine the moment of the force about point *O* .

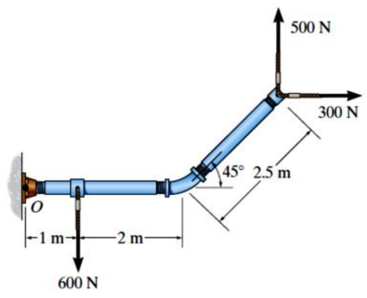












Chapter (3)

Equilibrium of a Rigid Body

1. OBJECTIVES

■ To develop the equations of equilibrium for a rigid body.
■ To introduce the concept of the free-body diagram for a rigid body.
■ To show how to solve rigid-body equilibrium problems using the equations of equilibrium.

2. Conditions for Rigid-Body Equilibrium

In this section, we will develop both the necessary and sufficient conditions for the equilibrium of the rigid body in Fig. 11. As shown, this body is subjected to an external force and couple moment system that is the result of the effects of gravitational, electrical, magnetic, or contact forces caused by adjacent bodies. The internal forces caused by interactions between particles within the body are not shown in this figure because these forces occur in equal but opposite collinear pairs and hence will cancel out, a consequence of Newton's third law.

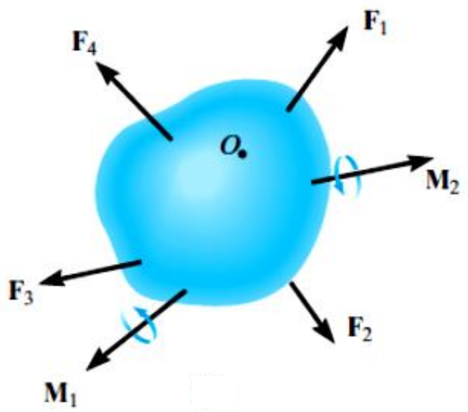


Fig. 11

3. Free-Body Diagrams

Successful application of the equations of equilibrium requires a complete specification of *all* the known and unknown external forces that act *on* the body. The best way to account for these forces is to draw a free-body diagram. This diagram is a sketch of the outlined shape of the body, which represents it as being *isolated* or "free" from its surroundings, i.e., a "free body." On this sketch it is necessary to show *all* the forces and couple moments that the surroundings exert *on the body* so that these effects can be accounted for when the equations of equilibrium are applied. *A thorough understanding of how to draw a free-body diagram is of primary importance for solving problems in mechanics.*

4. Support Reactions

Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule,

• If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.

• If rotation is prevented, a couple moment is exerted on the body.

For example, let us consider three ways in which a horizontal member, such as a beam, is supported at its end. One method consists of a *roller* or cylinder, Fig. 12 *(a)*. Since this support only prevents the beam from *translating* in the vertical direction, the roller will only exert a *force* on the beam in this direction, Fig. 12 *(b)*.

The beam can be supported in a more restrictive manner by using a *pin*, Fig. 12 *(c)*. The pin passes through a hole in the beam and two leaves which are fixed to the ground. Here the pin can prevent *translation* of the beam in *any direction* f, Fig. 12 *(d)*, and so the pin must exert a *force* **F** on the beam in this direction. For purposes of analysis, it is generally easier to represent this resultant force **F** by its two rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$, Fig. 12 *(e)*. If $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are known, then $\boldsymbol{F}$ and f can be calculated.

The most restrictive way to support the beam would be to use a *fixed support* as shown in Fig. 12 *(f)*. This support will prevent both *translation and rotation* of the beam. To do this a *force and couple moment* must be developed on the beam at its point of connection, Fig. 12 *(g)*. As in the case of the pin, the force is usually represented by its rectangular components $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$.

Table 1 lists other common types of supports for bodies subjected to coplanar force systems. (In all cases the angle u is assumed to be known.) Carefully study each of the symbols used to represent these supports and the types of reactions they exert on their contacting members.

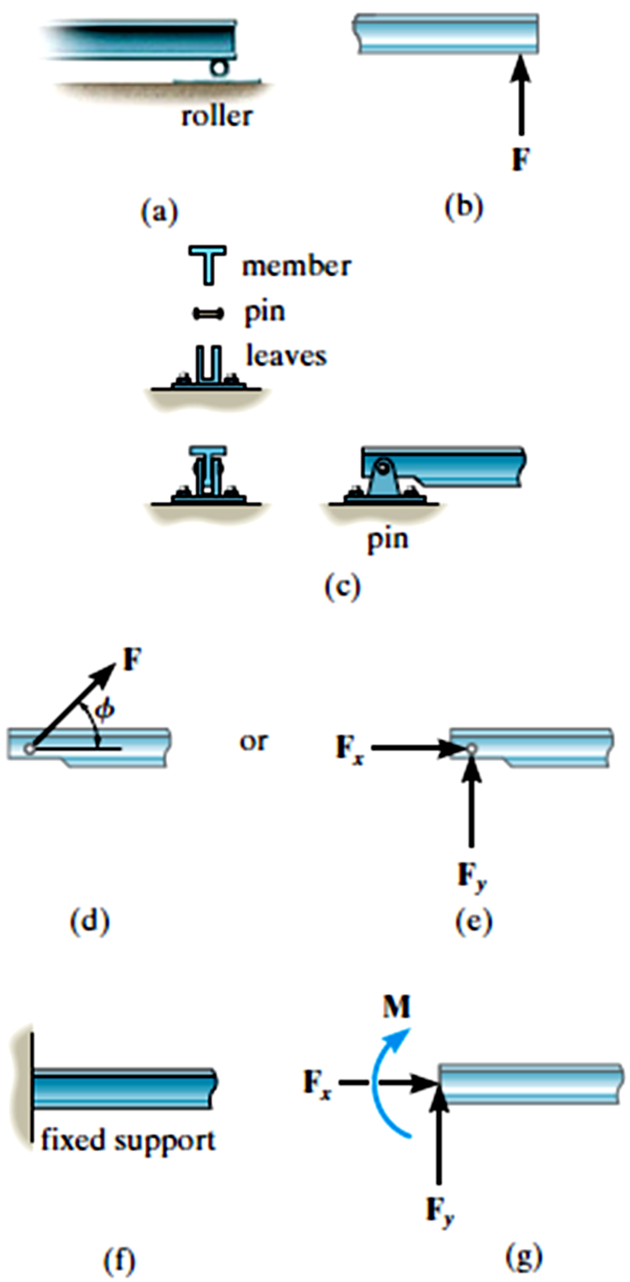
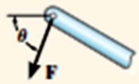
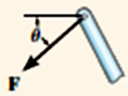
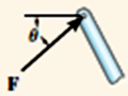
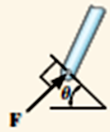
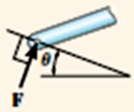
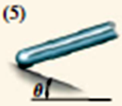
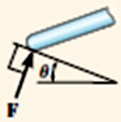
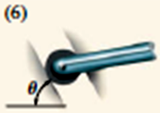
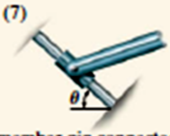
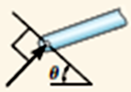
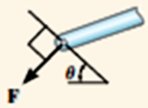

roller
F
(a)
(b)
member
pin
leaves
pin
(c)
F
ϕ
or
F_x
F_y
(d)
(e)
M
fixed support
F_x
F_y
(f)
(g)

Fig. 12.

TABLE -1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

5. Equations of Equilibrium

In Sec. 2 we developed the two equations which are both necessary and sufficient for the equilibrium of a rigid body, namely, $\sum \mathbf{F} = \mathbf{0}$ and $\sum \mathbf{M}_O = \mathbf{0}$. When the body is subjected to a system of forces, which all lie in the $x - y$ plane, then the forces can be resolved into their $\boldsymbol{x}$ and $\boldsymbol{y}$ components. Consequently, the conditions for equilibrium in two dimensions are

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$\sum M_O = 0$

Example: 1

Draw the free-body diagram of the uniform beam shown in Fig. 13. The beam has a mass of 100 kg. And determine the reactions moment and forces.

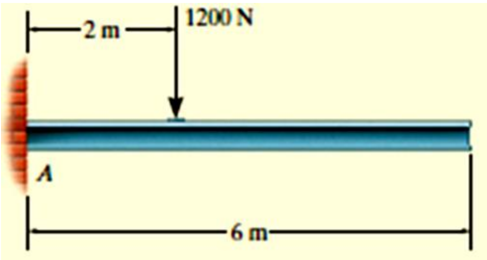


Fig. 13.

Solution:

The free-body diagram of the beam is shown in Fig. 14. Since the support at ***A*** is fixed, the wall exerts three reactions *on the beam*, denoted as A_x, A_y, and M_A. The magnitudes of these reactions are *unknown*, and their sense has been *assumed*. The weight of the beam, **W = 100(9.81) N = 981 N**, acts through the beam's center of gravity ***G***, which is ***3 m*** from ***A*** since the beam is uniform.

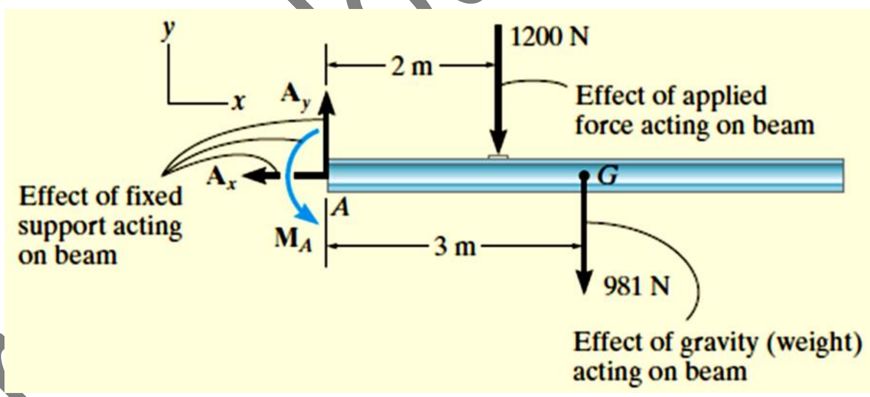


Fig. 14.

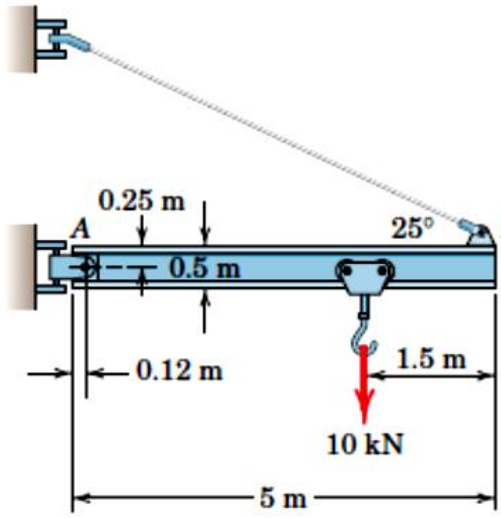
From the equilibrium conditions we get;

$\sum F_x = 0 \quad \therefore A_x = 0$

$\sum F_y = 0 \quad \therefore A_y = 1200 + 981 = 2181\ N$

$\sum M_O = 0 \ \therefore M_A = (981\ N \times 3\ m) + (1200\ N \times 2\ m) = 5343\ N.m$

Example: 2



Determine the magnitude ***T*** of the tension in the supporting cable and the magnitude of the force on the pin at ***A*** for the jib crane shown. The beam ***AB*** is a standard **0.5-m *I-beam*** with a mass of 95 kg per meter of length.

Solution:

Algebraic solution.

The system is symmetrical about the vertical *x-y* plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at ***A*** represented in terms of its two rectangular components. The weight of the beam is ***95(10³)(5)9.81 = 4.66 kN*** and acts through its center. Note that there are three unknowns A_x, A_y, and ***T***, which may be found from the three equations of equilibrium. We begin with a moment equation about ***A***, which eliminates two of the three unknowns from the equation. In applying the moment equation about *A*, it is simpler to consider the moments of the ***x***- and ***y***-components of **T** than it is to compute the perpendicular distance from **T** to ***A***. Hence, with the counterclockwise sense as positive we write

$[\sum M_A = 0]$ $(T\cos 25^o)0.25 + (T\sin 25^o)(5-0.12) - 10(5-1.5-0.12) - 4.66(2.5-0.12) = 0$

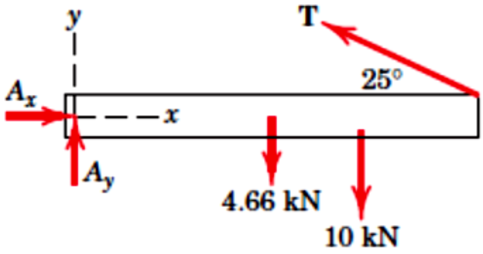
From which $T = 19.61\ kN$ ***Ans.***

Equating the sums of forces in the *x*- and *y*-directions to zero gives

$[\sum F_x = 0]$ $A_x - 19.61\cos 25^o = 0 \quad \therefore A_x = 17.77\ kN$

$[\sum F_y = 0]$ $A_y + 19.61\sin 25^o - 466 - 10 = 0 \quad \therefore A_y = 6.37\ kN$

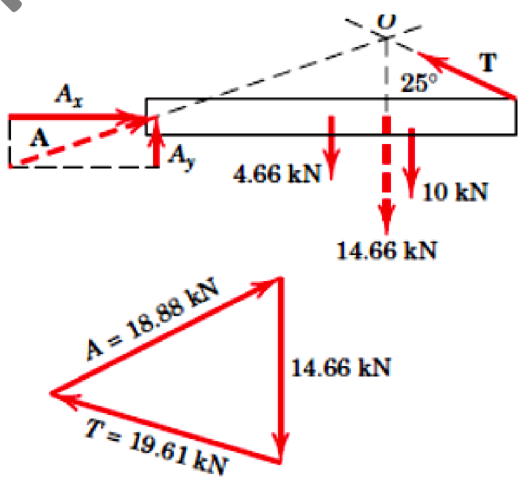
$\left|A = \sqrt{{A_x}^2 + {A_y}^2}\right|$ $A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88\ kN$ ***Ans.***



Free-body diagram

Graphical solution.

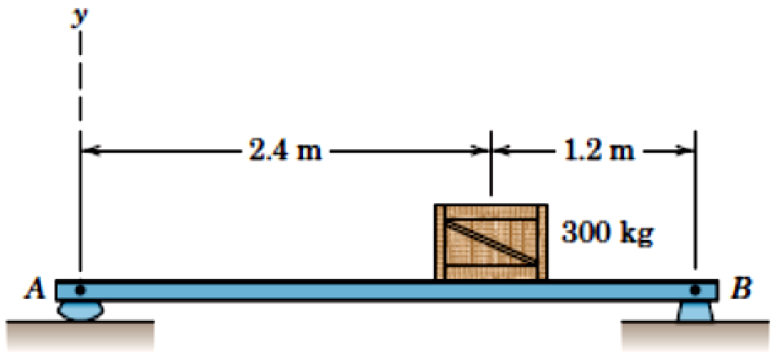
The principle that three forces in equilibrium must be concurrent is utilized for a graphical solution by combining the two known vertical forces of ***4.66*** and ***10 kN*** into a single ***14.66-kN*** force, located as shown on the modified free-body diagram of the beam in the lower figure. The position of this resultant load may easily be determined graphically or algebraically. The intersection of the ***14.66-kN*** force with the line of action of the unknown tension **T** defines the point of concurrency ***O*** through which the pin reaction **A** must pass. The unknown magnitudes of **T** and **A** may now be found by adding the forces head-to-tail to form the closed equilibrium polygon of forces, thus satisfying their zero-vector sum. After the known vertical load is laid off to a convenient scale, as shown in the lower part of the figure, a line representing the given direction of the tension **T** is drawn through the tip of the ***14.66-kN*** vector. Likewise, a line representing the direction of the pin reaction **A**, determined from the concurrency established with the free-body diagram, is drawn through the tail of the ***14.66-kN*** vector. The intersection of the lines representing vectors **T** and **A** establishes the magnitudes *T* and *A* necessary to make the vector sum of the forces equal to zero. These magnitudes are scaled from the diagram. The **x-** and ***y***-components of *A* may be constructed on the force polygon if desired.



Graphical solution

Example: 3

The uniform beam has a mass of ***50 kg*** per meter of length. Determine the reactions at the supports.



Solution:

From the free-body diagram we get

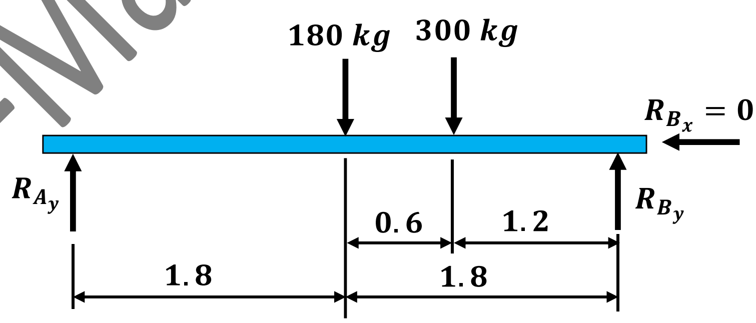
$[\sum F_x = 0] \quad \therefore R_{B_x} = 0$

$[\sum F_y = 0] \quad \therefore R_{A_y} + R_{B_y} = 300 + 180 = 480\ kN \qquad (1)$

$[\sum M_A = 0] \qquad (300 \times 2.4) + (180 \times 1.8) - \left(R_{B_y} \times 3.6\right) = 0$

$$R_{B_y} = \frac{(300 \times 2.4) + (180 \times 1.8)}{3.6} = 290\ kN$$

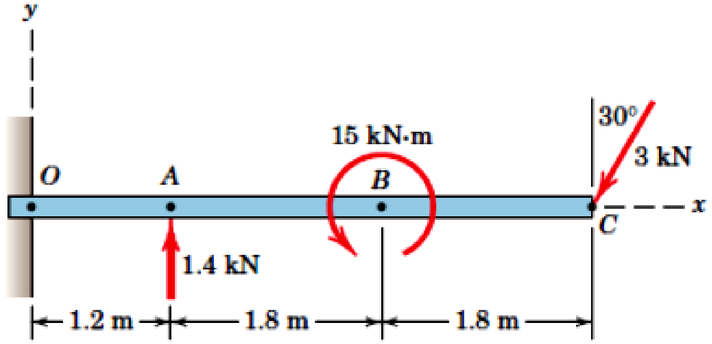
From Eqn. (1) $\quad \therefore R_{A_y} = 480 - 290 = 190\ kN$



Free-body diagram

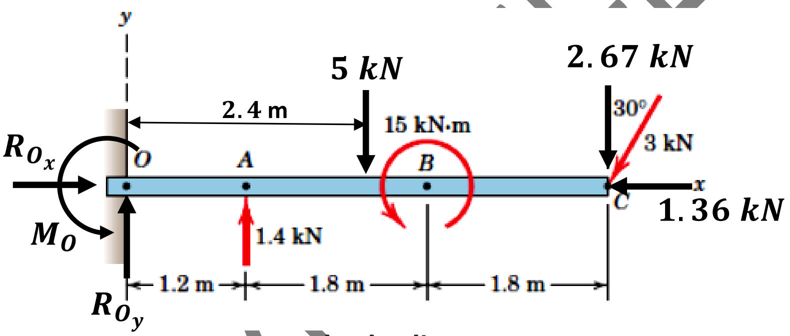
Example: 4

The 500-kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point O. The x-y plane is vertical.



Solution:

From the free-body diagram we get



Free-body diagram

$[\sum F_x = 0]$ $R_{O_x} - 1.36 = 0$ $\quad\therefore R_{O_x} = 1.36\ kN$

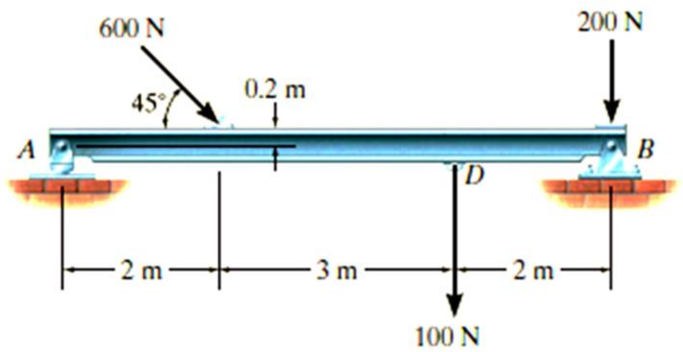
$[\sum F_y = 0]$ $\therefore R_{O_y} + 1.4 - 5 - 2.67 = 0$ $\quad\therefore R_{O_y} = 6.27\ kN$

$[\sum M_0 = 0]$ $(2.67 \times 4.8) + (5 \times 2.4) - 15 - (1.4 \times 1.2) - M_0 = 0$

$$\therefore M_0 = 8.136\ kN.m \quad CCW$$

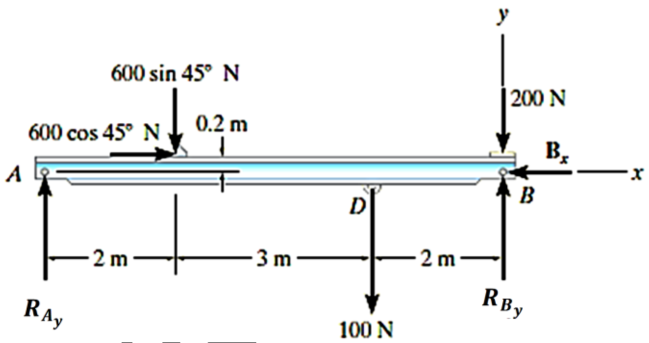
Example: 5

Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig.. Neglect the weight of the beam.



Solution:

From the free-body diagram we get



Equations of Equilibrium. Summing forces in the x direction yields

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 600 \cos 45^o - R_{B_x} = 0$$

$$\therefore R_{B_x} = 424\ N \qquad \textbf{Ans.}$$

A direct solution for $\boldsymbol{A_y}$ can be obtained by applying the moment equation $+\circlearrowleft \sum M_B = 0$ about point $\boldsymbol{B}$.

$$\circlearrowleft + \sum M_B = 0;$$

$$(100 \times 2) + (600 \sin 45^o \times 5) - (600 \cos 45^o \times 2) - \left(R_{A_y} \times 7\right) = 0$$

$$\therefore R_{A_y} = 319\ N \qquad \textbf{Ans.}$$

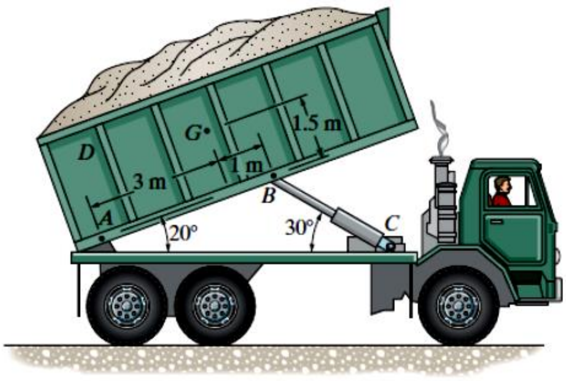
Summing forces in the y direction, using this result, gives

$$+\uparrow \sum F_y = 0; \qquad 319 - 600 \sin 45^o - 100 - 200 + R_{B_y} = 0$$

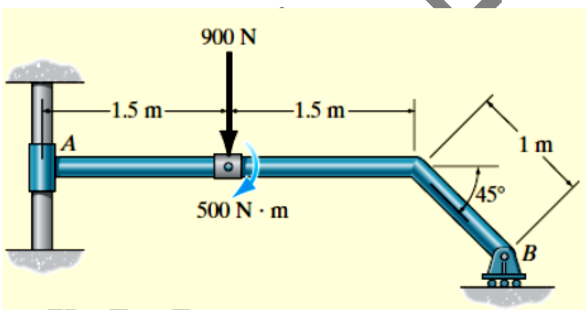
$$\therefore R_{B_y} = 405\ N \qquad \textbf{Ans}$$

FUNDAMENTAL PROBLEMS

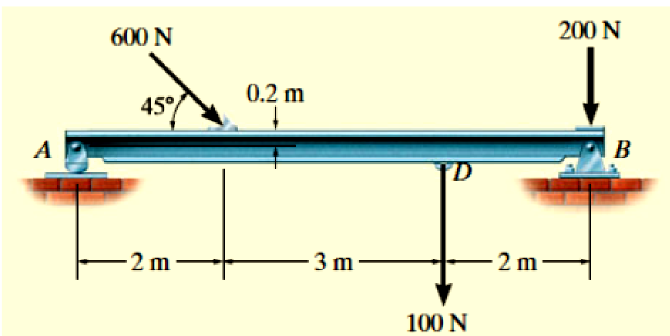
1. Draw the free-body diagram of the dumpster ***D*** of the truck, which has a mass of ***2.5 Mg*** and a center of gravity at ***G***. It is supported by a pin at ***A*** and a pin-connected hydraulic cylinder ***BC*** (short link). Determine the horizontal and vertical components



2. Determine the support reactions on the member. The collar at *A* is fixed to the member and can slide vertically along the vertical shaft.

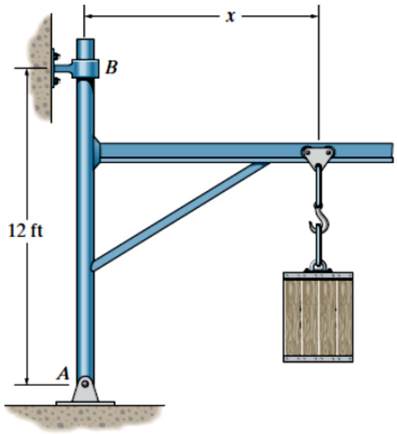


3. Determine the horizontal and vertical components of reaction on the beam caused by the pin at *B* and the rocker at *A* as shown in Figure. Neglect the weight of the beam.

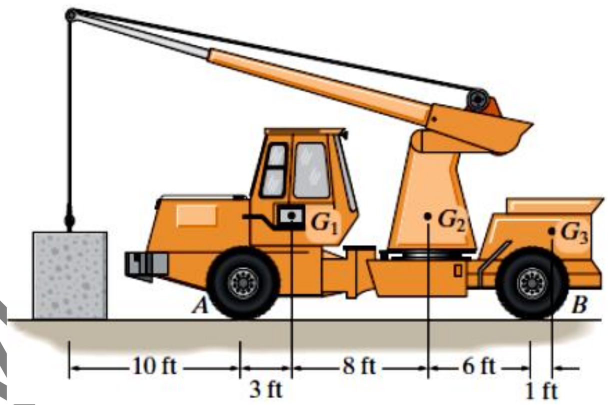


4. The jib crane is pin connected at ***A*** and supported by a smooth collar at ***B***. Determine the roller placement ***x*** of the ***5000-lb*** load so that it gives the

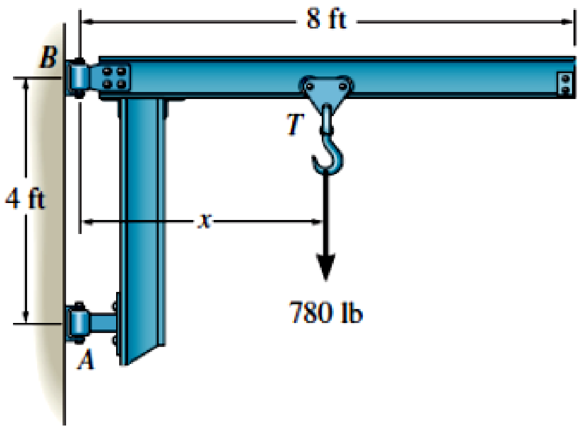
maximum and minimum reactions at the supports. Calculate these reactions in each case. Neglect the weight of the crane. Require ***4 ft*** $\le$ ***x*** $\le$ ***10 ft***.



5. The crane consists of three parts, which have weights of W_1 = 3500 lb, $W2$ = 900 lb, W_3 = 1500 lb and centers of gravity at G_1, G_2, and G_3, respectively. Neglecting the weight of the boom, determine (a) the reactions on each of the four tires if the load is hoisted at constant velocity and has a weight of 800 lb, and (b), with the boom held in the position shown, the maximum load the crane can lift without tipping over.



6. The cantilevered jib crane is used to support the load of ***780 lb***. If the trolley ***T*** can be placed anywhere between ***1.5 ft*** $\le$ ***x*** $\le$ ***7.5 ft***, determine the maximum magnitude of reaction at the supports ***A*** and ***B***. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at ***B*** supports a force in the vertical direction, whereas the one at ***A*** does not.



Chapter (4)

Friction

CHAPTER OBJECTIVES

- To introduce the concept of dry friction and show how to analyze the equilibrium of rigid bodies subjected to this force.
- To present specific applications of frictional force analysis on wedges, screws, belts, and bearings.
- To investigate the concept of rolling resistance

1. Characteristics of Dry Friction

Friction is a force that resists the movement of two contacting surfaces that slide relative to one another. This force always acts ***tangent*** to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

In this chapter, we will study the effects of ***dry friction***, which is sometimes called *Coulomb friction* since its characteristics were studied extensively by C. A. Coulomb in 1781. Dry friction occurs between the contacting surfaces of bodies when there is no lubricating fluid. *

2. Theory of Dry Friction.

The theory of dry friction can be explained by considering the effects caused by pulling horizontally on a block of uniform weight**W** which is resting on a rough horizontal surface that is *nonrigid or deformable,* Fig. 1 *(a).* The upper portion of the block, however, can be considered rigid. As shown on the free-body diagram of the block, Fig. 1 *(b),* the floor exerts an uneven *distribution* of both *normal force* ΔN_n and *frictional force* ΔF_n along the contacting surface.
For equilibrium, the normal forces must act *upward* to balance the block's weight **W,** and the frictional forces act to the left to prevent the applied force **P** from moving the block to the right. Close examination of the contacting surfaces between the floor and block reveals how these frictional and normal forces develop, Fig. 1 *(c*). It can be seen that many microscopic irregularities exist between the two surfaces and, as a result, reactive forces ΔR_n are developed at each point of contact. As shown, each reactive force contributes both a frictional component ΔF_n and a normal component ΔN_n.

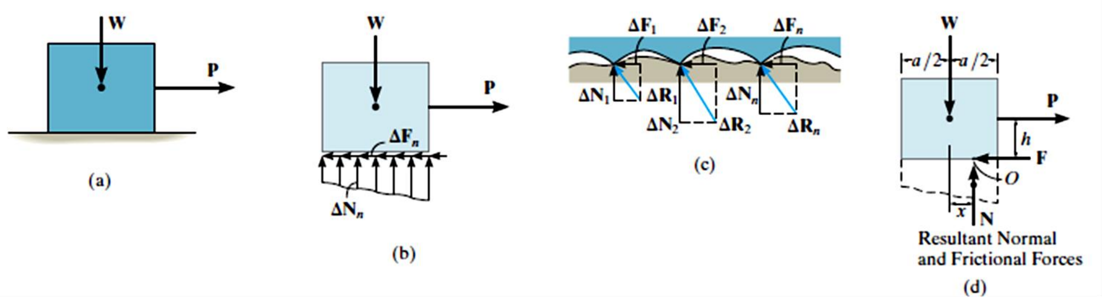


Fig. 1

Equilibrium. The effect of the *distributed* normal and frictional loadings is indicated by their *resultants* **N** and **F** on the free-body diagram, Fig. 1 *(d)*. Notice that **N** acts a distance x to the right of the line of action of **W,** Fig. 1 *(d)*. This location, which coincides with the centroid or geometric center of the normal force distribution in Fig. 1*(b)*, is necessary in order to balance the "tipping effect" caused by **P.** For example, if **P** is applied at a height h from the surface, Fig. 1 *(d)*, then moment equilibrium about point O is satisfied if $W_x = Ph$ **or** $x = Ph/W$.

Impending Motion. In cases where the surfaces of contact are rather "slippery," the frictional force **F** may *not* be great enough to balance **P,** and consequently the block will tend to slip. In other words, as P is slowly increased, F correspondingly increases until it attains a certain *maximum value* F_s, called the ***limiting static frictional force***, Fig. 1 *(e)*. When this value is reached, the block is in *unstable equilibrium* since any further increase in P will cause the block to move. Experimentally, it has been determined that this limiting static frictional force F_s is *directly proportional* to the resultant normal force N. Expressed mathematically,

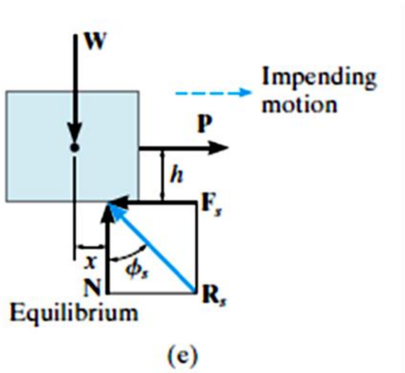


Fig. 1 (cont.)

$$F_s = \mu_s N \tag{1}$$

Where the constant of proportionality, $\boldsymbol{\mu_s}$ (mu "sub" *s*), is called the ***coefficient of static friction***.

Thus, when the block is on the *verge of sliding*, the normal force **N** and frictional force $\boldsymbol{F_s}$ combine to create a resultant $\boldsymbol{R_s}$, Fig. 1 *(e)*. The angle $\boldsymbol{\phi_s}$ (phi "sub" *s*) that $\boldsymbol{R_s}$ makes with ***N*** is called the *angle of static friction.*

From the figure,

$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s \qquad (2)$$

Typical values for $\boldsymbol{\mu_s}$ are given in Table 1. Note that these values can vary since experimental testing was done under variable conditions of roughness and cleanliness of the contacting surfaces. For applications, therefore, it is important that both caution and judgment be exercised when selecting a coefficient of friction for a given set of conditions. When a more accurate calculation of $\boldsymbol{F_s}$ is required, the coefficient of friction should be determined directly by an experiment that involves the two materials to be used.

Table 8–1 Typical Values for μ_s

Contact Materials	Coefficient of Static Friction (μ_s)
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Aluminum on aluminum	1.10–1.70

Motion. If the magnitude of ***P*** acting on the block is increased so that it becomes slightly greater than $\boldsymbol{F_s}$, the frictional force at the contacting surface will drop to a smaller value $\boldsymbol{F_k}$, called the ***kinetic frictional force***.

The block will begin to slide with increasing speed, Fig. 2 *(a)*. As this occurs, the block will "ride" on top of these peaks at the points of contact, as shown in Fig. 2 *(b)*. The continued breakdown of the surface is the dominant mechanism creating kinetic friction.

Experiments with sliding blocks indicate that the magnitude of the kinetic friction force is directly proportional to the magnitude of the resultant normal force, expressed mathematically as

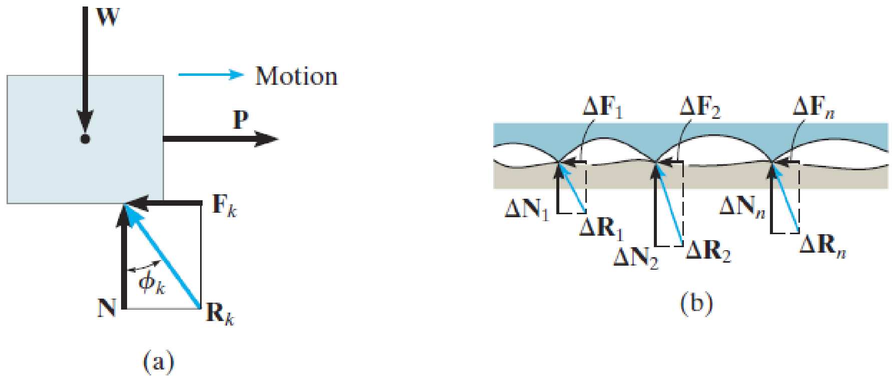


Fig. 2

$$F_K = \mu_k N \tag{3}$$

Here the constant of proportionality, $\boldsymbol{\mu_k}$, is called the ***coefficient of kinetic friction***. Typical values for m*k* are approximately **25 percent *smaller*** than those listed in Table 1 for $\boldsymbol{\mu_s}$.

As shown in Fig. 2 *(a)*, in this case, the resultant force at the surface of contact, $\boldsymbol{R_k}$, has a line of action defined by $\boldsymbol{\phi_k}$. This angle is referred to as the ***angle of kinetic friction***, where

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}\mu_k \tag{4}$$

By comparison, $\phi_s \geq \phi_k$.

The above effects regarding friction can be summarized by referring to the graph in Fig. 3, which shows the variation of the frictional force ***F*** versus the applied load ***P***. Here the frictional force is categorized in three different ways:

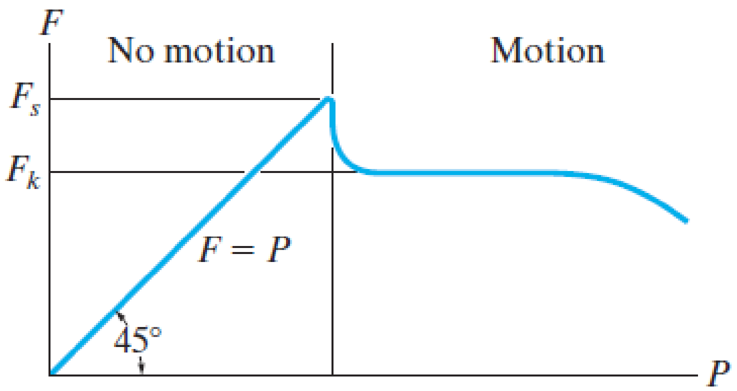


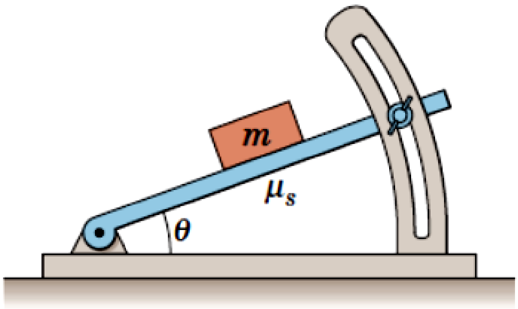
Fig. 3

• ***F*** is a ***static frictional force*** if equilibrium is maintained.
• ***F*** is a ***limiting static frictional force*** $\boldsymbol{F_s}$ when it reaches a maximum value needed to maintain equilibrium.
• ***F*** is a ***kinetic frictional force*** $\boldsymbol{F_k}$ when sliding occurs at the contacting surface.
Notice also from the graph that for very large values of ***P*** or for high speeds, aerodynamic effects will cause $\boldsymbol{F_k}$ and likewise $\boldsymbol{\mu_k}$ to begin to decrease.

Characteristics of Dry Friction. As a result of *experiments* that pertain to the foregoing discussion, we can state the following rules which apply to bodies subjected to dry friction.
• The frictional force acts *tangent* to the contacting surfaces in a direction *opposed* to the *motion* or tendency for motion of one surface relative to another.
• The maximum static frictional force $\boldsymbol{F_s}$ that can be developed is independent of the area of contact, provided the normal pressure is not very low nor great enough to severely deform or crush the contacting surfaces of the bodies.
• The maximum static frictional force is generally greater than the kinetic frictional force for any two surfaces of contact. However, if one of the bodies is moving with a *very low velocity* over the surface of another, $\boldsymbol{F_k}$ becomes approximately equal to $\boldsymbol{F_s}$, i.e., $\boldsymbol{\mu_s \approx \mu_k}$.
• When *slipping* at the surface of contact is *about to occur*, the maximum static frictional force is proportional to the normal force, such that $\boldsymbol{F_s = \mu_s N}$.
• When *slipping* at the surface of contact is *occurring*, the kinetic frictional force is proportional to the normal force, such that $\boldsymbol{F_k = \mu_k N}$.

Example: 1

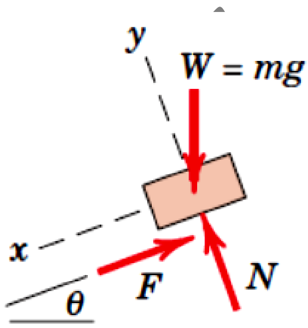
Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



Solution:

The free-body diagram of the block shows its weight $W = mg$, the normal force N, and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present.

Equilibrium in the x- and y-directions requires



$$[\sum F_x = 0] \quad mg \sin\theta - F = 0 \qquad \therefore F = mg \sin\theta$$

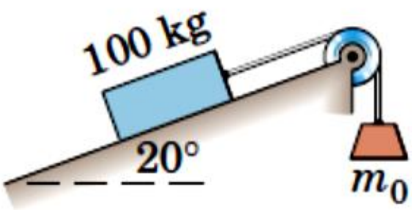
$$[\sum F_y = 0] \quad -mg \cos\theta + N = 0 \qquad \therefore N = mg \cos\theta$$

Dividing the first equation by the second gives $F/N = \tan\theta$. Since the maximum angle occurs when $F = F_{max} = \mu_s N$, for impending motion we have

$$\mu_s = \tan\theta_{max} \qquad \text{or} \qquad \theta_{max} = \tan^{-1}\mu_s$$

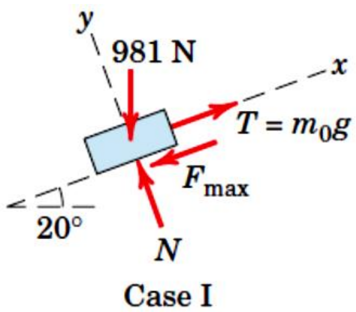
Example: 2

Determine the range of values which the mass m_0 may have so that the ***100-kg*** block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is ***0.30***.



Solution:

The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down



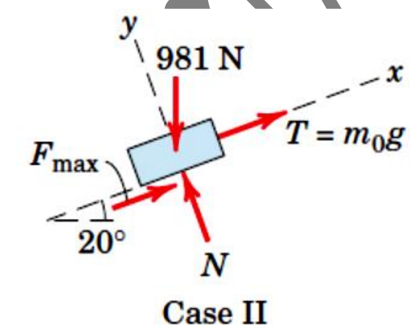
the plane, as shown in the free-body diagram of the block for ***Case I*** in the figure. With the weight ***mg = 100(9.81) = 981 N***, the equations of equilibrium give

$$[\sum F_y = 0] \quad N - mg\ \cos 20^o = 0 \qquad \therefore N = \ 922\ N$$

$$F_{max} = \mu_s N \qquad\qquad \therefore F_{max} = 0.30 \times 922 = 277\ N$$

$$[\sum F_x = 0] \quad m_0 9.81 - 277 - 981\ \sin 20^o = 0 \quad \therefore m_0 = \ 62.4\ kg$$ **Ans.**

The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for ***Case II***. Equilibrium in the ***x-direction*** requires



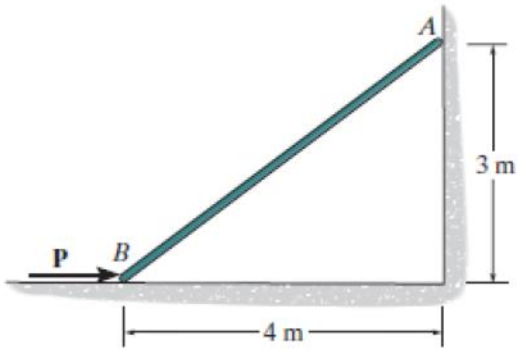
$$[\sum F_x = 0] \quad m_0 9.81 + 277 - 981\ \sin 20^o = 0 \quad \therefore m_0 = \ 6.01\ kg$$ **Ans.**

Thus, m_0 may have any value from ***6.01*** to ***62.4 kg***, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{max} and ***N*** be concurrent with the ***981 N*** weight and the tension ***T***.

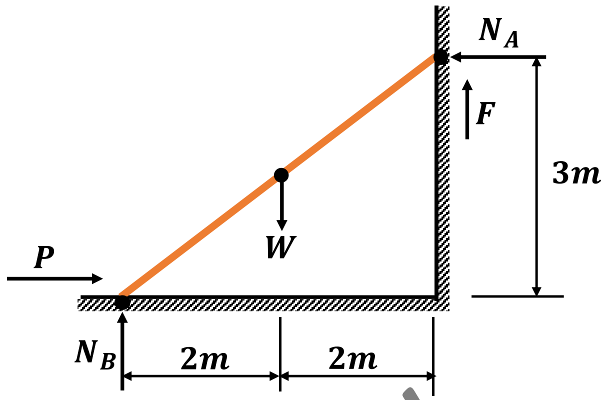
Example: 3

Determine the minimum force ***P*** to prevent the ***30-kg*** rod ***AB*** from sliding. The contact surface at ***B*** is smooth, whereas the coefficient of static friction between the rod and the wall at ***A*** is $\mu_s = 0.2$.



Solution:

From free body diagram we get;



Free body diagram

Bu using the equilibrium conditions

$1 - \sum M_B = 0$

$\therefore (N_A \times 3) + (F \times 4) - (W \times 2) = 0$

$F = \mu_S N_A = 0.2 N_A$

$\therefore (3N_A) + (0.2 \times 4N_A) = (2W)$

$$\therefore N_A = \frac{2W}{[3 + (0.2 \times 4)]} = \frac{2 \times 300 \times 9.81}{[3 + (0.2 \times 4)]} = 154.89\ N$$

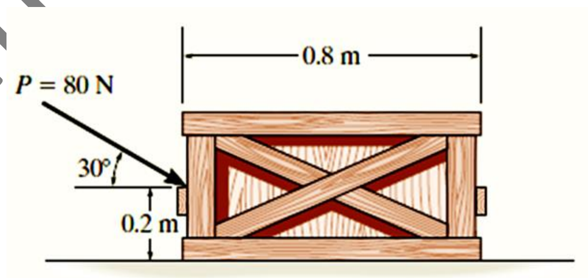
$2 - \sum F_x = 0$

$\therefore P - N_A = 0$

$\therefore P = 154.89\ N$

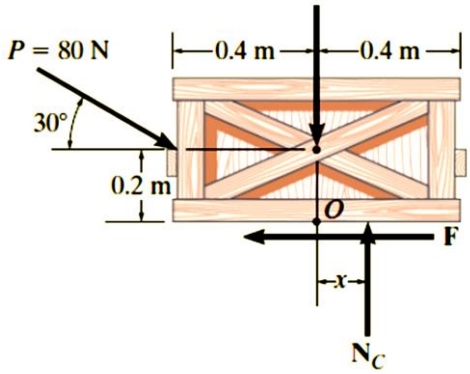
Example: 4

The uniform crate shown in Figure has a mass of ***20 kg***. If a force ***P = 80 N*** is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\boldsymbol{\mu_s = 0.3}$.



Solution:

From Free-Body Diagram. the *resultant* normal force N_C must act a distance x from the crate's center line in order to counteract the tipping effect caused by **P**. There are *three unknowns*, ***F***, N_C, and ***x***, which can be determined strictly from the ***three* equations** of equilibrium.



Equations of Equilibrium.

$\sum F_x = 0 \qquad \therefore 80 \cos 30^o - F = 0$

$\therefore F = 80 \cos 30^o = 69.3\ N$

$\sum F_y = 0 \qquad \therefore -80 \sin 30^o + N_C - 196.2 = 0$

$\sum M_0 = 0 \qquad \therefore 80 \sin 30^o\ (0.4) - 80 \cos 30^o\ (0.2) + N_C(x) = 0$

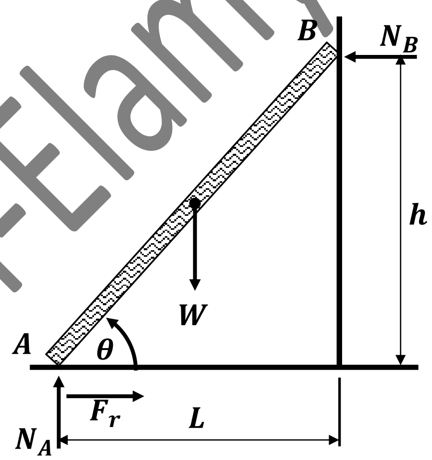
$\therefore N_C = 236.2\ N$ and x = -0.00908 m = -9.08 mm

Since *x* is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since *x* < 0.4 m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236.2\ N) = 70.9\ N$.

Since *F* = 69.3 N < 70.9 N, the crate will *not slip*, although it is very close to doing so.

Example: 5

The uniform ***10-kg*** ladder shown in Figure rests against the smooth wall at ***B***, and the end ***A*** rests on the rough horizontal plane for which the coefficient of static friction is $\boldsymbol{\mu_s = 0.3}$. Determine the angle of inclination $\boldsymbol{\theta}$ of the ladder and the normal reaction at ***B*** if the ladder is on the verge of slipping.



Free body diagram

Answer:

from the free body diagram, we have

by using the equilibrium conditions

$\sum F_x = 0 \quad \therefore F_r - N_B = 0 \qquad \therefore F_r = N_B \qquad (1)$

$\sum F_Y = 0 \quad \therefore N_A - W = 0 \qquad \therefore N_A = W \qquad (2)$

From Eqn. (2) $\quad \therefore N_A = 10 \times 9.81 = 98.1\ N$

$\because F_r = \mu_s N_A = 0.3 \times 98.1 = 29.43\ N$

From Eqn. (1) $\quad \therefore N_B = F_r = 29.43\ N$

$\sum M_A = 0 \quad \therefore N_B \times h - W \times (L/2) = 0$

$\therefore N_B \times h = W \times (L/2) \qquad (3)$

From the triangle

$h = 4 \sin\theta$ *and* $\quad L = 4 \cos\theta$

From Eqn. (3)

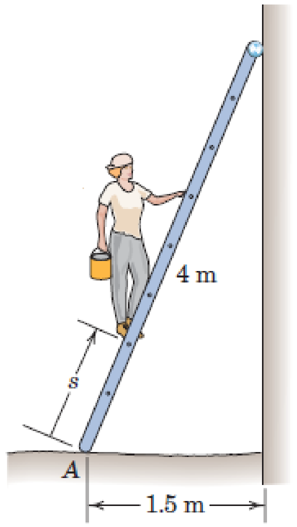
$\therefore N_B \times 4 \sin\theta \;= W \times (4 \cos\theta / 2)$

$\therefore 29.43\ \sin\theta \;= 49.05 \cos\theta$

$$\therefore \theta = \tan^{-1}\frac{49.05}{29.43} = 59.04^0$$

Example: 6

Determine the distance ***s*** to which the ***90-kg*** painter can climb without causing the ***4-m*** ladder to slip at its lower end ***A***. The top of the ***15-kg*** ladder has a small roller, and at the ground the coefficient of static friction is ***0.25***. The mass center of the painter is directly above her feet.



Solution:

$$L = \sqrt{4^2 - 1.5^2} = 3.71\ m$$

$$\theta = \cos^{-1}\left(\frac{1.5}{4}\right) = 67.98^o$$

By using the equilibrium conditions, we get

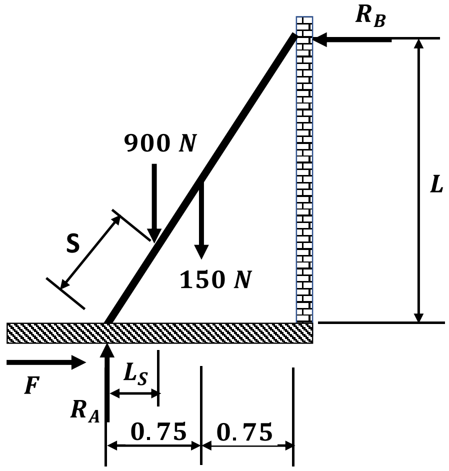
$$\sum F_x = 0 \quad \therefore F - R_B = 0 \qquad \therefore R_B = F = \mu R_A \qquad (1)$$

$$\sum F_y = 0 \quad \therefore R_A - 900 - 150 = 0 \qquad \therefore R_A = 900 + 150 = 1050\ N \qquad (2)$$

From Eqs. (1) and (2)

$$\therefore R_B = 0.25 \times 1050 = 262.5\ N$$

$$\sum M_A = 0 \qquad \therefore (R_B \times L) - (150 \times 0.75) - (900 \times L_S) = 0$$



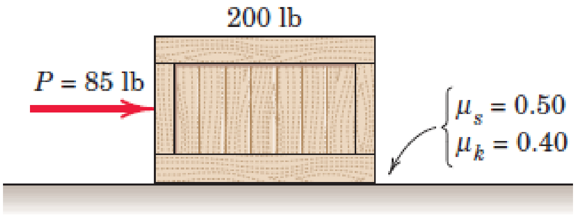
$$\therefore L_S = \frac{(262.5 \times 3.71) - (150 \times 0.75)}{900} = 0.957\ m$$

$$\because \cos\theta = \frac{L_S}{s}$$

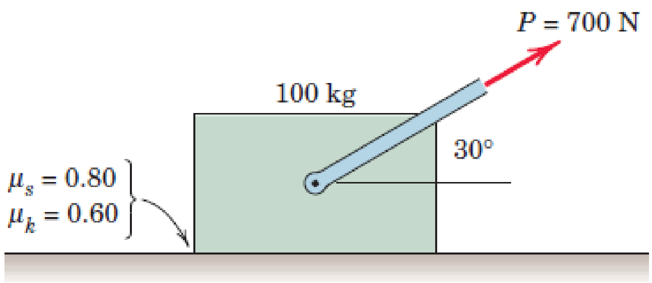
$$\therefore s = \frac{L_S}{\cos\theta} = \frac{0.957}{\cos 67.98^o} = 2.55\ m$$

Problems

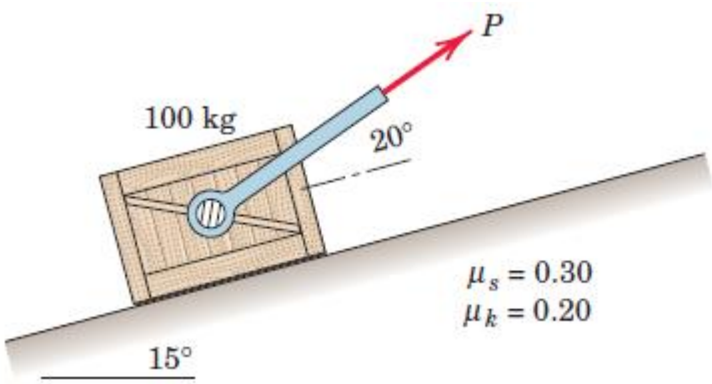
1. The 85-lb force P is applied to the 200-lb crate, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the crate.



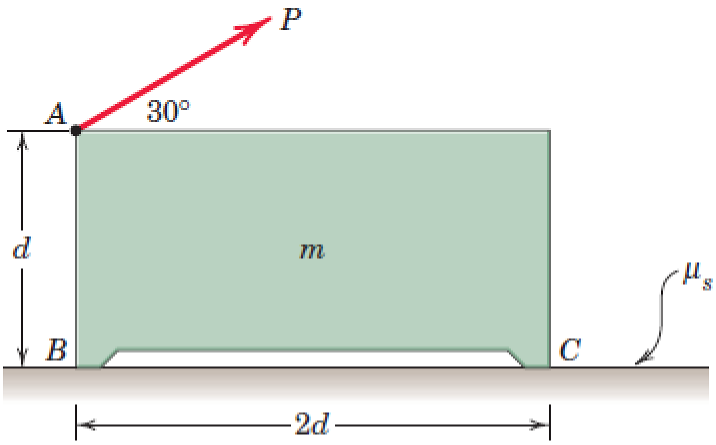
2. The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block.



3. The coefficients of static and kinetic friction between the 100-kg block and the inclined plane are 0.30 and 0.20, respectively. Determine (*a*) the friction force F acting on the block when P is applied with a magnitude of 200 N to the block at rest, (*b*) the force P required to initiate motion up the incline from rest, and (*c*) the friction force F acting on the block if P = 600 N.



4. The magnitude of force P is slowly increased. Does the homogeneous box of mass m slip or tip first? State the value of P which would cause each occurrence. Neglect any effect of the size of the small feet.

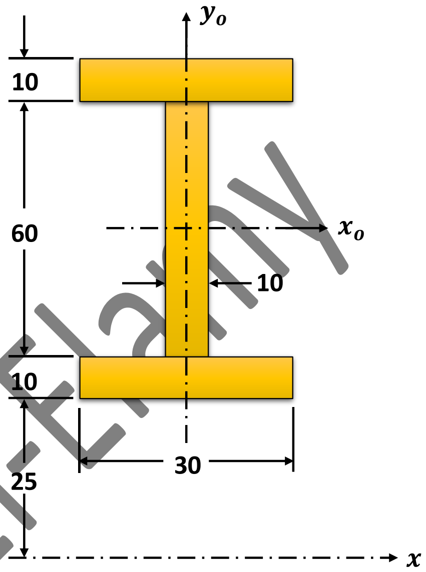


Chapter (5)

Area Moments of Inertia

Solved problems:

Example:1



Determine the moment of inertia of the cross-sectional area of the channel with respect to the x_o and y_o **axis**. Also Determine the moment of inertia of the cross-sectional area of the channel with respect to the ***x axis.***

Solution:

First, the geometric shape should be divided into a set of geometric shapes.

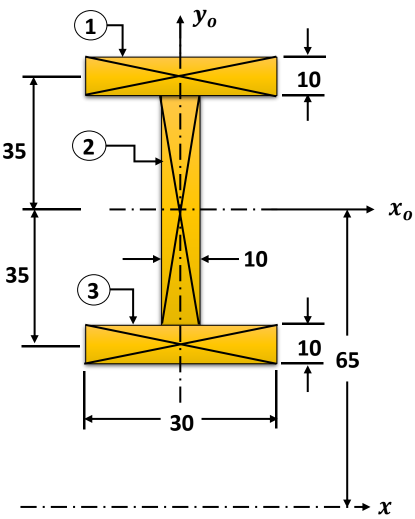
From the figure can be divided into three parts.

1. **With respect to the x_o axis**

Part: 1

$$I_{x_{o_1}} = \frac{b_1 h_1^3}{12} + A_1 d_1^2 =$$

$$= \frac{30 \times 10^3}{12} + (30 \times 10) \times 35^2 = 370000\ mm^4$$



Part: 2

$$I_{x_{o_2}} = \frac{b_2 h_2^3}{12} = \frac{10 \times 60^3}{12} = 180000\ mm^4$$

Part: 3

$$I_{x_{o_3}} = \frac{b_3 h_3^3}{12} + A_3 d_3^2 =$$

$$= \frac{30 \times 10^3}{12} + (30 \times 10) \times 35^2 = 370000\ mm^4$$

$$I_{x_o} = I_{x_{o_1}} + I_{x_{o_2}} + I_{x_{o_3}} = 370000 + 180000 + 370000 = 920000\ mm^4$$ ***Ans.***

The radius of gyration:

$$A_t = A_1 + A_2 + A_3 = (30 \times 10) + (60 \times 10) + (30 \times 10) = 1200\ mm^2$$

$$k_{x_o} = \sqrt{\frac{I_{x_o}}{A}} = \sqrt{\frac{920000}{1200}} = 27.69\, m$$

2. With respect to the y_o axis

Part: 1

$$I_{y_{o_1}} = \frac{\mathrm{h_1 b_1^3}}{12} = \frac{10 \times 30^3}{12} = 22500\, mm^4$$

Part: 2

$$I_{y_{o_2}} = \frac{\mathrm{h_2 b_2^3}}{12} = \frac{60 \times 10^3}{12} = 5000\, mm^4$$

Part: 3

$$I_{y_{o_3}} = \frac{\mathrm{h_3 b_3^3}}{12} = \frac{10 \times 30^3}{12} = 22500\, mm^4$$

$$I_{y_o} = I_{y_{o_1}} + \mathrm{I}_{y_{o_2}} + \mathrm{I}_{y_{o_3}} = 22500 + 5000 + 22500 = 50000\, mm^4$$ ***Ans.***

The radius of gyration:

$$k_{y_o} = \sqrt{\frac{I_{y_o}}{A}} = \sqrt{\frac{50000}{1200}} = 6.45\, m$$

3. With respect to the x axis

$$I_x = I_{x_o} + A_t d_x^2$$

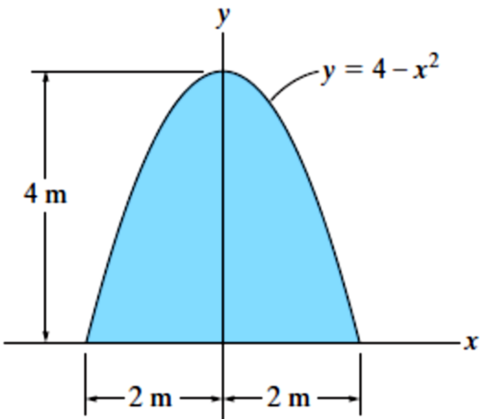
$$\therefore I_x = 920000 + (1200 \times 65^2) = 5990000\, mm^4$$ ***Ans.***

The radius of gyration:

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5990000}{1200}} = 70.65\, m$$

Example:2

Determine the moment of inertia of the area about the *y* axis.



Solution:

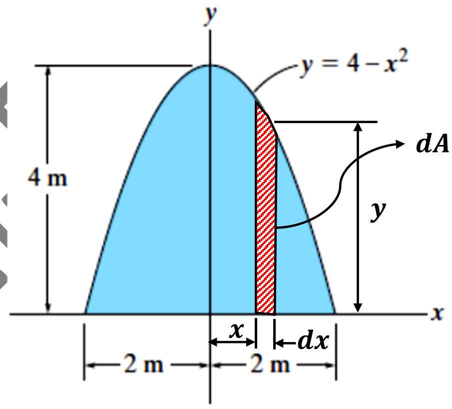
A differential element of area that is *parallel* to the *y* axis, as shown in Figure, is chosen for integration. Since this element has a thickness *dy* and intersects the curve at the *arbitrary point* (x, *y)*, its area is *dA* = y *dx*. Hence, integrating with respect to *y* , from *x* = 0 to *x* = 2 m, yields.

$$I_y = \int x^2\, dA$$

$$dA = ydx = (4 - x^2)dx$$

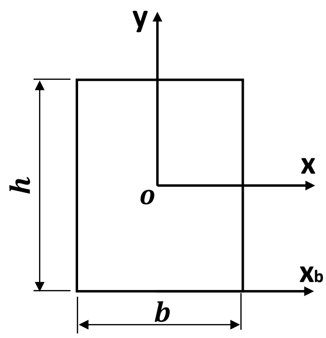
$$I_y = 2\int_0^2 (4x^2 - x^4)\, dx = 2\left|4\frac{x^3}{3} - \frac{x^5}{5}\right|_0^2$$

$$= 2\left|4\frac{2^3}{3} - \frac{2^5}{5}\right|_0^2 = 2 \times \frac{64}{15} = 8.53\, m^4$$



Example:3

Determine the moment of inertia for the rectangular area shown in Fig. 10–5 with respect to (a) the centroidal *x*- axis, (b) the axis x_b passing through the base of the rectangle, and (c) the pole or *z*-axis perpendicular to the *x-y* plane and passing through the centroid ***O***.

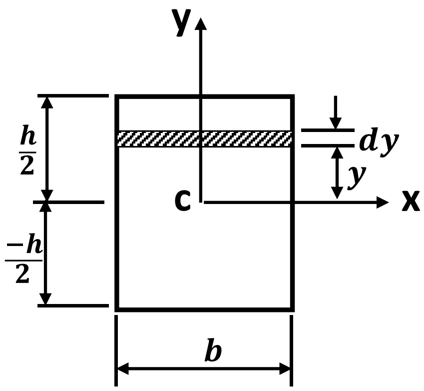


Solution:

a. *At the centroidal x-axis*

$$dI_x = \int y^2 dA$$

Where: $dA = b.dy$



$$\therefore I_x = \int_{-h/2}^{h/2} y^2\, b.dy = b\int_{-h/2}^{h/2} y^2\, dy = \left|\frac{y^3}{3}\right|_{-h/2}^{h/2} = \frac{1}{12}bh^3$$

$$\therefore I_y = \frac{1}{12}hb^3$$

b. *At the x_b-axis*

$$\therefore I_{xb} = I_x + Ad^2$$

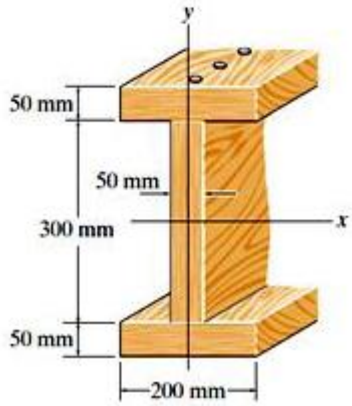
$$\therefore I_{xb} = \frac{1}{12}bh^3 + b.h.\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3$$

c. At the z-axis

$$\therefore J_C = I_x + I_Y = \frac{1}{12}bh^3 + \frac{1}{12}hb^3 = \frac{1}{12}[hb^3 + bh^3]$$

Example:3

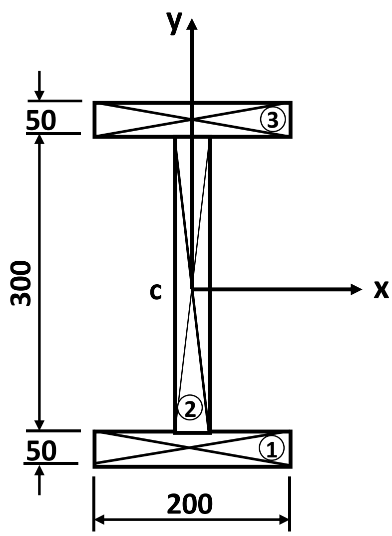
Determine the moment of inertia of the cross-sectional area of the channel with respect to the *y* axis. Determine the *radius of gyration* of an area about y axis



<u>**Solution:**</u>

$$dI_y = x^2 dA$$

$$\therefore I_y = \int_{x1}^{x2} x^2 dA$$



<u>*Part: (1)*</u>

$$I_{y_1} = \frac{1}{12}hb^3 = \frac{1}{12}(50)(200)^3 = 33333333.3\ mm^4$$

<u>*Part: (2)*</u>

$$I_{y_2} = \frac{1}{12}hb^3 = \frac{1}{12}(300)(50)^3 = 3125000\ mm^4$$

<u>*Part: (3)*</u>

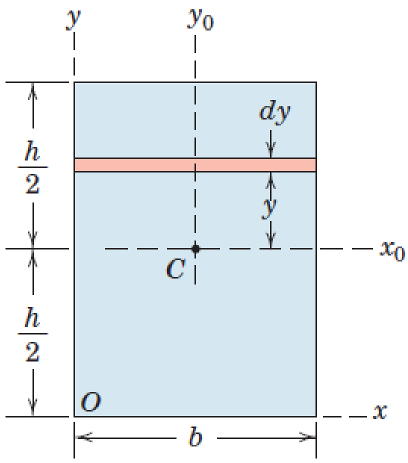
$$I_{y_3} = \frac{1}{12}hb^3 = \frac{1}{12}(50)(200)^3 = 33333333.3\ mm^4$$

$$\therefore I_y = I_{y_1} + I_{y_2} + I_{y_3} = 33333333.3 + 3125000 + 33333333.3 = 69791666.6\ mm^4$$
$$= 69.8 \times 10^6\ mm^4$$

<u>**The *radius of gyration*:**</u>

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{69791666.6}{35000}} = 44.65\ m$$

Example:4



Determine the moments of inertia of the rectangular area about the centroidal x_0- and y_0-axes, the centroidal polar axis z_0 through ***C***, the *x*-axis, and the polar axis *z* through ***O***.

Solution:

For the calculation of the moment of inertia I_{x0} about the x_0-**axis**, a horizontal strip of area ***b dy*** is chosen so that all elements of the strip have the same *y*-coordinate. Thus,

$$I_{x_0} = \int y^2 \, dA$$

Where a horizontal strip of area ***dA = b dy***

$$I_{x_0} = \int_{-h/2}^{+h/2} y^2 \, b \, dy = b \left| \frac{y^3}{3} \right|_{-h/2}^{h/2} = \frac{1}{12} b h^3$$

By interchange of symbols, the moment of inertia about the centroidal y_0-***axis*** is

$$I_{y_0} = \int_{-b/2}^{+b/2} x^2 \, h \, dx = h \left| \frac{x^3}{3} \right|_{-b/2}^{b/2} = \frac{1}{12} h b^3$$

The centroidal polar moment of inertia is

$$I_{z_0} = I_{x_0} + I_{y_0}$$

$$= \frac{1}{12} b h^3 + \frac{1}{12} h b^3 = \frac{1}{12} b h (b^2 + h^2) = \frac{1}{12} A (b^2 + h^2)$$

By the parallel-axis theorem the moment of inertia about the ***x-axis*** is

$$I_x = I_{x_0} + A d^2$$

$$= \frac{1}{12} b h^3 + (b.h) \left(\frac{h}{2} \right)^2 = \frac{1}{3} b h^3 = \frac{1}{3} A h^2$$

We also obtain the polar moment of inertia about *O* by the parallel-axis theorem, which gives us

$$I_z = I_{z_0} + A d^2$$

$$= \frac{1}{12} A (b^2 + h^2) + (b.h) \left[\left(\frac{h}{2} \right)^2 + \left(\frac{h}{2} \right)^2 \right] = \frac{1}{3} b h (b^2 + h^2) = \frac{1}{3} A (b^2 + h^2)$$